Statistical Approach to Radioactive Target Detection and Location via Wireless Sensor Networks

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Abstract—The detection of materials or devices for nuclear or radiological weapons of mass destruction is fundamentally important to national safety and security, and detection technologies are necessary both to find and to verify the location of those materials or devices. To fulfill this task, this paper presents a statistical approach to the detection and location of radioactive target via wireless sensor networks. The statistical approach includes hypothesis test of the existence and the point or interval estimation of the location of the target. Theoretical and empirical properties have been studied. The result showed that the proposed approach was efficient and effective in the detection and location of radioactive target.

Keywords—asymptotical distribution; likelihood ratio test; maximum likelihood estimates; signal plus background model; radiation and radioactive materials; wireless sensor network.

I. INTRODUCTION

Since the world trade center tragedies on September 11, 2001, the United States as well as many other countries has acknowledged the high possibility of terrorist attacks using weapons of mass destruction (WMD), which arise the need to improve the national security level to defense the terrorism [20]. The detection of materials or devices for nuclear or radiological WMD is fundamentally important to the improvement of the national security level. Detection technologies are necessary both to find and to verify the location of those materials or devices. When considering the development of these technologies, detection and location of nuclear radioactive signals are never far from center stage. A nuclear detection involves using nuclear detectors to obtain data, converting data into usable information through algorithm, and acting on that information through concepts of operations [18]. Under this framework, a wireless sensor network (WSN) can be used as the system to obtain data. A statistical method can be used as an algorithm to convert data into usable information. Therefore, the aim of this research is to develop a statistical approach to the detection and location of nuclear materials or devices based on WSN data.

The importance of this research has been highlighted by the recent Fukushima nuclear power station disasters caused by the earthquake in Japan on March 11, 2001. After the earthquake, radioactive materials have been released from the Fukushima containment vessels which have made serious nuclear pollution in the World. Currently, it is still too early to judge the final outcomes of the nuclear crisis that continues to Japan. This tragedy uncovers a huge safety and security problems that exist in all of the nuclear power plants in the World. In order to improve the safety and security level for nuclear power stations, it is important to build a quick and accurate detection system for the leakage of nuclear materials on nuclear units. The method developed in this paper will provide a theoretical base for such a system. The idea of the method is to test the statistical significance of radiation signals from a potential nuclear radioactive target by assuming radiation detectors are used as sensor nodes in a WSN. Today, many passive and active, fixed and portable radiation detectors are available commercially.

Recently, WSN is an emerging technology for monitoring physical processes with a densely distributed network of wireless nodes. Currently, using WSN is becoming less expensive since recent advances in wireless communications and electronics have enabled the development of low cost, lower-power, multifunctional sensor nodes that are small in size and efficiently communicate in short distance [17]. These tiny sensor nodes which consist of sensing, data processing, and communicating components, enhanced the ideas of WSN. Technologies of WSN have wide applications in sensing building and structures monitoring [26], traffic control [9], oceanographic data collection and pollution monitoring [1], and chemical source localization [24].

An essential application of WSN is to detect and locate targets over an area of interest. There are substantial amount of work on WSN for target detection [3], [5], [6], [7], [8], [16]. A general question for a WSN is how to efficiently integrate the available information from individual sensors to reach a global decision about the presence of a target over a monitoring area. The past work on signal detection with sensor networks can be categorized into two groups of methods: decision fusion and value fusion methods, respectively. In decision fusion, each sensor makes its own binary decision and then the network will make a consensus by fusing all decisions [4], [9], [16]. In value fusion, the sensor collects measurements and the network will make a decision by fusing the collected values. Because radiation is given by counts, it is more reasonable to believe that value fusion is more appropriate to use for nuclear radioactive target detection. Therefore in this paper, we focus on the methods for a value fusion WSN.
Suppose a value fusion WSN has been used to detect and locate a nuclear radioactive target. Assume the sensor nodes are radiation detectors. A radiation detector, also known as a particle detector, is a device used to detect, track, and identify high-energy particles emitted from radioactive materials, including neutrons, alpha particles, beta particles, and gamma rays. The observed radiation count received by a detector is a mixture of the signal emitted from the radioactive source and the background emitted from natural radionuclides [13]. In this model, neither the location nor the strength of the radioactive target is known. The classical approach is to assume a specific statistical model for the signal and frame it as a hypothesis test of the null hypothesis \( H_0 : \{ \text{no target is present in the study area} \} \) vs the alternative \( H_1 : \{ \text{there is a target in this area} \} \) [16]. Following this idea, we propose to test \( H_0 : \{ \text{the signal strength is zero} \} \) vs \( H_1 : \{ \text{the signal strength is positive} \} \). Because the radiation signal strength is inversely proportional to the distance between the detector and the radioactive target, but the background strength is homogeneously distributed over the study area [12], the optimal statistical approach is possible to detect and locate the potential nuclear radioactive target simultaneously.

The statistical approach will be introduced in the rest of this paper. In Section II, we briefly review the physical background for nuclear radioactive isotopes. In Section III, we propose our detection and location methods for nuclear radioactive targets based on statistical estimation and testing procedures. Because the detection method is nonstandard in statistics [10], in Section IV we propose a numerical method to access the null distribution of the test statistic. In Section V, we display our Monte Carlo simulation evaluations. In Section VI, we present our conclusion.

II. PHYSICAL BACKGROUND

Nuclear weapons or nuclear power plants contain fissile materials, which can sustain chain reactions. All fissile materials are radioactive and all nuclear detection technologies are designed to detect emissions from the decay of radioactive nuclides. The primary long-range observable from nuclear materials are gamma rays and neutrons, which have mean free paths of the order of hundreds of meters in air [19]. Even though gamma rays or neutrons do not have an electrical charge, they can produce electrical charges in certain detectors so that they can be measured by energy and counts [18].

There are several basic ways to detect radioactive materials based on the usage of radiation detectors. Among those, passive detection is the preferred technique because of its simplicity and safety. In passive detection, radioactive signals can be detected by measuring neutrons or gamma rays emitted from the radioactive materials. The count of neutrons or gamma rays follows a Poisson distribution with a certain intensity rate (per second), which varies from isotope to isotope depending on the halflife and the types of radiation emitted during radioactive decay. Several well-known radioactive isotopes can be found in [12], [18].

As a detector moves always from the radioactive target, the expected rate of neutrons or gamma rays decreases inversely with the square of the distance [12]. Let \( \omega = (\omega_1, \omega_2, \omega_3) \) be the location of the nuclear radioactive target and \( \bar{a} = (a_1, a_2, a_3) \) be the location of radiation detector. Let

\[
r(\omega) = \| \omega - \bar{a} \| = \sqrt{(\omega_1 - a_1)^2 + (\omega_2 - a_2)^2 + (\omega_3 - a_3)^2}
\]

be their Euclidean distance. Then, the radiation count received by the detector follows a homogeneous Poisson process with intensity rate equal to \( A \nu_s/(4\pi r^2(\omega)) \) (per second), where \( \nu_s \) is the surface radiation rate (per second) from the target, and \( A \) and \( e \) are the area and the efficiency of the detector respectively. Suppose the time of the detection period is \( T \) (seconds). Then, the total number of signal count received by the detector follows a Poisson distribution with expected value equal to \( T A \nu_s/(4\pi r^2(\omega)) \).

In addition to the target signal, the detector also receives background radiation. It has been known that the neutron background is due to cosmic rays and increases with altitude and geomagnetic latitude and the gamma-ray background is due to radionuclides in soils and rock. The background is independent of the signal. It is usually homogeneous in a short time period but may vary significantly in a long time period. They are especially high during the maximum phase of the solar activity cycle [12], [13], [18]. Therefore, in the detection with duration period \( T \) (seconds), the number of background radiation counts received by the detector follows a Poisson distribution with expected value equal to \( T A \nu_s \). Let \( Y \) be the total radiation count observed by the detector. Then, \( Y \) is the combination of the signal and background, with

\[
y \sim \text{Poisson}(T A \nu_s + \frac{\nu_s}{4\pi r^2(\omega)}).
\]

The unknown parameters contained in Model (1) includes the signal intensity \( \nu_s \), background intensity \( \nu_b \), and location parameter \( \omega \in \mathbb{R}^3 \).

This statistical model given by Equation (1) is called the signal plus background model [14], [21], [25]. In this model, a signal is combined with a background so that the total observation is their combination. The presence of the nuclear radioactive target is given by the case when \( \nu_s > 0 \), which leads to a statistical hypotheses testing problem as

\[
H_0 : \nu_s = 0 \quad \text{vs} \quad H_1 : \nu_s > 0.
\]

The classical approach to address this problem assumes only one detector is used. Here we assume multiple detectors are used and each is deployed at a different location so that they form a WSN. In this case, the background intensity rates are the same but the signal intensity rates are different as they depend on the distances to the target. By comparing the spatial distribution of the observed counts from all of the detectors, our method can simultaneously detect and locate a hidden nuclear radioactive target. This detail of this method will be discussed in the next Section.
III. THE STATISTICAL APPROACH

Assume a value fusion WSN has $m$ detectors (used for sensor nodes), which are deployed at $a_i = (a_{i1}, a_{i2}, a_{i3})$ for $i = 1, \ldots, m$ respectively. Let $r_i = r_i(\omega) = \|\omega - a_i\|$ be the Euclidean distance between the target and the $i$-detector, where $\omega = (\omega_1, \omega_2, \omega_3)$ is the location of the radioactive target. Denote $\xi_i = A_i e_i$, where $A_i$ is the area and $e_i$ is the efficiency of the $i$-th detector respectively. Let $y_i$ be the total number of radiation counts observed by the $i$-th detector during the detection with duration period $T$ (seconds). Then, the statistical model can be proposed as

$$y_i \sim \text{Poisson}(T\xi_i(v_b + \frac{\nu_s}{4\pi r_i^2})), i = 1, \ldots, m. \quad (3)$$

The unknown parameters contained in Model (3) are $v_b$, $\nu_s$ and $\omega$. Based on Model (3), the detection problem is interpreted by the hypothesis testing problem given by Equation (2). The location problem is interpreted as the estimation and confidence interval for $\omega$ in Model (3), which is only accessed when $H_0$ is rejected.

We propose a statistical method to test the significance of the null hypothesis. Our method is based on the famous likelihood ratio test, which is accessed by the likelihood ratio statistic [15]. The likelihood ratio statistic is defined by the ratio of likelihood functions under $H_0 \cup H_1 : \nu_s \geq 0$ and under $H_0 : \nu_s = 0$ respectively. When the test is significant, the estimate and the confidence interval for $\omega$ can be accessed by the asymptotical normality of the maximum likelihood estimator (MLE). In order to derive the likelihood ratio test statistic, we need to compute the maximum of the likelihood functions under $\nu_s \geq 0$ and $\nu_s = 0$ respectively. In order to derive the estimate and confidence interval for $\omega$, we need to derive the limiting distribution of the MLE of $\omega$. This will be introduced in the remaining part of this Section.

Straightforwardly, the loglikelihood function is

$$\ell(\nu_s, v_b, \omega) = \sum_{i=1}^m y_i \log[T\xi_i(v_b + \frac{\nu_s}{4\pi r_i^2})] - T \sum_{i=1}^m \xi_i(v_b + \frac{\nu_s}{4\pi r_i^2}) - \sum_{i=1}^m \log(y_i!) \quad (4)$$

Under $H_0 : \nu_s = 0$, $\ell(\nu_s, v_b, \omega)$ in Equation (4) becomes

$$\ell_0(v_b) = \sum_{i=1}^m y_i \log(T\xi_i v_b) - T v_b \xi_+ - \sum_{i=1}^m \log(y_i!), \quad (5)$$

where $\xi_+ = \sum_{i=1}^m \xi_i$. Because $\ell_0(v_b)$ only depends on $v_b$, it is not necessary to estimate $\omega$ under $H_0 : \nu_s = 0$.

By comparing Equations (4) and (5), we find that the location $\omega$ is not present in (5). Therefore, this problem is nonstandard because the classical loglikelihood ratio test does not possess it usually asymptotic null distribution. In order to well-define the loglikelihood ratio test, we first propose a conditional test statistic for a given $\omega$ and then maximize the conditional test statistic for all possible $\omega$. This method has been previously used in [10].

Suppose $\omega$ is pre-selected. Then, Equation (4) only contains parameters $\nu_s$ and $v_b$. In this case, the testing problem becomes standard. Therefore, the loglikelihood ratio test can be formulated by the conditional test statistic given by

$$\Lambda(\omega) = 2[\ell(\hat{\nu}_s, \hat{v}_b, \omega) - \ell_0(\hat{v}_b)] \quad (6)$$

where $\hat{v}_b = \sum_{i=1}^m Y_i/(T \xi_i)$ is the MLE of $v_b$ under $H_0 : \nu_s = 0$, and $\hat{\nu}_s$ and $\hat{v}_b$ are the conditional MLE of $\nu_s$ and $v_b$ under $H_0 \cup H_1 : \nu_s \geq 0$ respectively.

There is no analytic solution for $\hat{\nu}_s$ and $\hat{v}_b$. We propose a Newton-Raphson algorithm to compute their values. This algorithm needs to be developed carefully because $\ell(\nu_s, v_b, \omega)$ may have more than one local maxima. Because of the limited page of this paper, we omit the introduction of the algorithm.

Because $\omega$ is unknown, we assume it belongs to a set $D \subseteq R^3$. Then, the loglikelihood ratio test statistic is defined by

$$\Lambda = \sup_{\omega \in D} \Lambda(\omega). \quad (7)$$

The null hypothesis $H_0 : \nu_s = 0$ is rejected if $\Lambda$ is large. When the null distribution of $\Lambda$ is derived, the $p$-value of $\Lambda$ can be easily computed by the upper quantile function of the null distribution. If the $p$-value of $\Lambda$ is less than a pre-selected significance level $\alpha$, the test is significant; otherwise, the test is not significant.

When the test is significant, we need to locate the radioactive source. Since the problem becomes standard under $\nu_s > 0$, the location $\omega$ of the radioactive target can be easily derived by its asymptotical normality. Let $(\hat{\nu}_s, \hat{v}_b, \hat{\omega})$ be the MLE of $(\nu_s, v_b, \omega)$. Then, the $100(1 - \alpha)$% elliptical confidence region for $\omega$ is

$$C_{\alpha}(\omega) = \{ \omega : \frac{1}{T}(\omega - \hat{\omega})'I(\hat{\nu}_s, \hat{v}_b, \hat{\omega})(\omega - \hat{\omega}) \leq \chi^2_{\alpha,3} \}, \quad (8)$$

where $\chi^2_{\alpha,3}$ is the upper $\alpha$ quantile of $\chi^2_3$ distribution, and $I(\hat{\nu}_s, \hat{v}_b, \hat{\omega})$ is a $3 \times 3$ Fisher Information matrix given by

$$I_{j,k} = 4\hat{v}_b^2 \sum_{i=1}^m \xi_i(a_{ij} - \hat{\omega}_j)(a_{ik} - \hat{\omega}_k)/(\hat{\nu}_s r_i^2(\hat{\omega}) + \hat{v}_b r_i^2(\omega)), j, k = 1, 2, 3. \quad (9)$$

IV. THE NULL DISTRIBUTION OF $\Lambda$

Because the location parameter $\omega$, which is also called the nuisance parameter in this Section, disappears under $H_0 : \nu_s = 0$ in Model (3), the test given by Equation (2) is nonstandard because the classical likelihood ratio test (i.e. $\Lambda$ defined in (7)) does not process its usual $\chi^2$ asymptotic null distribution. Asymptotic analysis of such a testing problem for one-dimensional nuisance parameter case based on a set of regularity conditions has been theoretically studied by [10], [11]. General asymptotic analysis of the testing problem for high-dimensional nuisance parameter has been theoretically studied by [2]. However, none of them has provided a way to numerically compute the null distribution. Based on the specific formulation of the nuclear radioactive signal, it is possible to derive a numerical method to compute the null distribution. We summarize the method into the following theorem.
**Theorem 1:** Suppose $\xi_i$ and $\nu_0$ are fixed under $H_0 : \nu_0 = 0$ in Model 3. Then as $T \to \infty$, $\Lambda$ asymptotically converges to max$_{\omega \in D} \left[ \max[0, Z(\omega)] \right]^2$ in distribution, where $A$ is the domain of $\omega$ and $Z(\omega)$ a Gaussian random field with $V[Z(\omega)] = 1$ for all $\omega \in D$ and

$$
\rho(\omega, \omega') = \text{Cov}(Z(\omega), Z(\omega'))
$$

$$
\xi_+ c_2(\omega, \omega') - c_1(\omega) c_1(\omega')
$$

$$
[\xi_+ c_2(\omega, \omega) - c_1(\omega)]^{1/2} [\xi_+ c_2(\omega', \omega') - c_1(\omega')]^{1/2},
$$

for all $\omega, \omega' \in D$, where $c_1(\omega) = \sum_{i=1}^{m} \xi_i / r_i^2(\omega)$ and $c_2(\omega, \omega') = \sum_{i=1}^{m} \xi_i / [r_i^2(\omega) r_i^2(\omega')]$.

**Proof:** Let $Z(\omega)$ be the limiting distribution of $\tilde{\nu}_{s,\omega} / \sigma(\tilde{\nu}_s)$, where $\sigma^2(\tilde{\nu}_s)$ is the asymptotic variance of the $\tilde{\nu}_{s,\omega}$. Then $Z(\omega)$ is a Gaussian random field with $V[Z(\omega)] = 1$. From the method of M-estimation ([22], Page 52), we have

$$
\sqrt{T} \left( \begin{pmatrix} \tilde{\nu}_{s,\omega} \\ \tilde{\nu}_{b,\omega} \\ \tilde{\nu}_{b,\omega}' \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right) \Rightarrow N(0, V_{\omega,\omega'}),
$$

as $T \to \infty$ under the null hypothesis, where

$$
V_{\omega,\omega'} = \begin{pmatrix} C_{\omega}^{-1} & C_{\omega}^{-1} \tilde{C}_{\omega,\omega'} C_{\omega}^{-1} & C_{\omega}^{-1} \tilde{C}_{\omega,\omega'} \end{pmatrix}
$$

with

$$
C_{\omega} = \frac{1}{\nu_0} \begin{pmatrix} c_2(\omega, \omega) & c_1(\omega) \\ c_1(\omega) & \xi_+ \end{pmatrix},
$$

and

$$
\tilde{C}_{\omega,\omega'} = \frac{1}{\nu_0} \begin{pmatrix} c_2(\omega, \omega') & c_1(\omega') \\ c_1(\omega') & \xi_+ \end{pmatrix}.
$$

The covariance of $Z(\omega)$ and $Z(\omega')$ for any $\omega, \omega' \in A$ can be derived, which is exactly equal to $\rho(\omega, \omega')$ as we have displayed in the Theorem. Because Model (3) is a parametric model, $\tilde{\nu}_{s,\omega}$ converges to $Z(\omega)$ in distribution as $T \to \infty$ ([22], Page 271). For a given $\omega \in D$ as $T \to \infty$, the limiting distribution of $\Lambda(\omega)$ is asymptotically equivalent to $\max(0, \tilde{\nu}_{s,\omega}^2 / \sigma^2(\tilde{\nu}_s))$. This implies the conclusion of the Theorem. \hfill \Box

Theorem 1 implies that the limiting null distribution of $\Lambda$ only depends on the location of detectors $\alpha_i$ and the product of the area and efficacy of detector $\xi_i = A_i \xi_1$ for $i = 1, \ldots, m$. Therefore, it can be numerically derived when $\omega$ and $\xi_i$ are determined, which implies the null distribution of $\Lambda$ does not vary when the system is set up. Because the asymptotic null distribution of $\Lambda$ does not depend on the background parameter $\nu_0$, the null distribution does not need to be recomputed even if $\nu_0$ changes over a long time period.

**V. MONTE CARLO SIMULATION ANALYSIS**

In our numerical analysis, we assumed the WSN had 100 radiation sensors. They were identical and deployed at the $10 \times 10$ lattice. The radioactive target was installed at $(5.5, 5.5)$. Consequently, the distance between the radioactive target and the $i$ detector was

$$
r_i = r_i(\omega) = ||\alpha_i - \omega|| = \sqrt{(a_{i1} - 5.5)^2 + (a_{i2} - 5.5)^2},
$$

for $i = 1, \ldots, m$ with $(a_{i1}, a_{i2})$ be the $i$-th lattice points of the $10 \times 10$ lattice. The radiation count $y_i$ received by the $i$-th detector then has the distribution

$$
y_i \sim \text{Poisson}(T \xi_i (\nu_0 + \nu_s / 4\pi r_i^2)).
$$

We fixed $\nu_0 = 1$ in all of our simulations. We assumed the sensors were identical so that $\xi_i$ were all the same. To make our simulation simpler, we assumed $\xi_i = 1$ for all $i = 1, \ldots, 100$. In order to make the problem easy to be understood, we assumed the third dimension of the deployed sensors were all 0 and the nuclear radioactive target was hidden on the same plane of the deployed sensors. That is, we assumed $\omega_3 = a_{i3} = 0$ for all $i$.

At the beginning, we numerically studied the null distribution of $\Lambda$ for $T = 5, 10, 20, 50, 100$. For each selected $T$, we randomly generated $10^5$ datasets from Model (9) by letting $\nu_s = 0$ and derived the value of $\Lambda$ for each generated dataset. Therefore, for each selected $T$, we had $10^5$ values of $\Lambda$. Based on those $10^5$ values of $\Lambda$, we computed the null distribution of $\Lambda$ by using the method of empirical distribution ([22]). The quantile functions of the empirical distributions are given in Table I. It can be seen that the null distribution of $\Lambda$ for those selected $T$ were almost identical. In the following of our simulation, we treated $T = 100$ as the limiting distribution of $\Lambda$. Therefore based on the result of Table I, we would reject the null hypothesis $H_0 : \nu_s = 0$ and claim the existence of a nuclear radioactive target in the study area if $\Lambda > 11.62$ when the significance level $\alpha = 0.05$, or if $\Lambda > 15.15$ when the significance level $\alpha = 0.01$.

After the null distribution was derived, we further evaluated the test statistic $\Lambda$ based on the behavior of its power function by using significance level $\alpha = 0.05$. When the test was significant, we then evaluated the location estimates based on the behavior of the mean square error of $\omega$. The computation was based on Monte Carlo simulations with repetition 1000 runs.

When dataset was generated, we computed the value $\Lambda$. We claimed that there was no radioactive target in the study area if the value was less than or equal to 11.62. If the value was greater than 11.62, we claimed there existed a radioactive target in the area. In this case, we derived the MLE $\hat{\omega}$ of $\omega$.

<table>
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<td>15.13</td>
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</tr>
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and then computed

$$
||\hat{\omega} - \omega||^2 = (\hat{\omega}_1 - 5.5)^2 + (\hat{\omega}_2 - 5.5)^2.
$$

The power function of $\Lambda$ was derived by the percentage of the rejections. The MSE of $\hat{\omega}$ was derived by the average of $||\hat{\omega} - \omega||^2$ when the test was significant. We let $\nu_s$ vary from 0 to 2 with step increment 0.5. The result is given in Table II.

It can be seen that the power function of $\Lambda$ increased as $\nu_s$ became stronger. The type I error probability was displayed by the case when $\nu_s = 0$. As we expected, the power function was close to 0.05 when $\nu_s = 0$. In addition, the table also showed that the MSE of $\hat{\omega}$ approached to 0 as $\nu_s$ became stronger, which indicated the radiation signal can be successfully located as $T$ became large. In addition, we have found that the results were almost identically if $\sqrt{T\nu_s/\nu_b}$ kept a constant. Therefore, Table II can represent a large group of scenarios.

Rather than the setting of $\alpha_i$ and $\xi_i$ that we have displayed in Tables I and II, we also studied other setting cases. Because the quantile function of the null distributions of $\Lambda$, the power function of $\Lambda$ and the MSE of $\hat{\omega}$ were very similar, we did not display them in this paper.

VI. CONCLUSION

In this paper, we have proposed a statistical detection and location method for nuclear radioactive target. The method proposed in this paper was based on value fusion WSN. This is an extension of our previous work [23]. In this paper, we have derived the limiting null distribution of $\Lambda$ and studied its robustness. Comparing the result in Table I, we found the null distribution of $\Lambda$ cannot be approximated by a $\chi^2$ distribution because the quantile function of $\chi^2$ distribution are very different from the result that we have displayed in Table I. This will also be extensively studied in our future research. Comparing with methods using a single detector, our method can simultaneously detect and locate nuclear radioactive targets. The idea presented in this paper will be extensively used to real world nuclear safety and security problems.

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