Homework 7

STAT 514

1. An experiment was run to determine whether different colored candles (red, white, blue, yellow) burn at different speeds. Each experimenter collected four observations on each color in a random order, and "experimenter" was used as a blocking factor. The design was a general complete block design with $v = 4, k = 16, b = 4$, and $s = 4$. The resulting burning times (in seconds) are shown in Table 17.21 in the book and can be downloaded at http://deanvossdraguljic.ietsandbox.net/DeanVossDraguljic/SAS-data/candle.sas.

(a) Analyze the experiment as though the experimenters represent a random sample from a large population of people who might use these candles in practice. Use a two-way mixed model with interaction where the experimenters are blocks with random effects and the interactions of block and color are random as well. The color has fixed effects. Provide the 95% simultaneous confidence intervals for all pairwise comparisons of color using Tukey's method.

(b) Provide the SAS code for 1(a).

(c) In 1(a), suppose we do not treat these experimenters as a random sample from a large population and consequently the model does not have random effects. Provide the 95% simultaneous confidence intervals for all pairwise comparisons of color using Tukey's method.

(d) Provide the SAS code for 1(c)

(e) Which model provides shorter confidence intervals? Give a justification if you could.

(f) If a two-way main-effects model was used in 1(a) and 1(c), do you think the two models (one with random effects and one without) would produce the simultaneous confidence intervals of different lengths?

2. Suppose in a completely randomized designs experiment there are 3 crossed factors A, B and C. Their levels are $a = 4$, $b = 3$ and $c = 3$. Each treatment is replicated $r = 2$ times. The following model is applied to the experiment:

$$Y_{ijkt} = \mu + \alpha_i + \beta_j + \gamma_k + (\beta\gamma)_{jk} + \epsilon_{ijkt}, i = 1, 2, 3, 4; j, k = 1, 2, 3,$$

where ϵ_{ijkt}'s are i.i.d. $N(0, \sigma^2)$, α_i's represent the fixed effects of A, β_j's and γ_k's are the random effects of Factors B and C, respectively, and $(\beta\gamma)_{jk}$'s are the random interaction effects. Furthermore, assume all random terms are independent and

$$\beta_j \sim N(0, \sigma_B^2), \gamma_k \sim N(0, \sigma_C^2), (\beta\gamma)_{jk} \sim N(0, \sigma_{BC}^2).$$

Partial SAS output is provided below

Dependent Variable: y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	4.20183285	0.38198480	0.35	0.9712
Error	60	66.31427369	1.10523789		
Corrected Total	71	70.51610654			

R-Square	Coeff Var	Root MSE	y Mean
0.059587	10.43951	1.051303	10.07043

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	3	0.95832853	0.31944284	0.29	0.8331
B	2	0.34514265	0.17257132	0.16	0.8558
C	2	0.10863270	0.05431635	0.05	0.9521
B*C	4	2.78972897	0.69743224	0.63	0.6423

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Be careful to choose the right denominator for the following questions.

(a) The F-ratio for testing the hypothesis that A has equal main effects is _____.

(b) The degree of freedom of the denominator for the hypothesis testing in the previous question is _____.

(c) The F-ratio for testing the hypothesis that B has no effects is _____.

(d) The degree of freedom of the denominator for the hypothesis testing in the previous question is _____.