Homework 6

STAT 514

1. The water heating experiment is described on page 338. The purpose of the experiment was to determine the best way to heat 3 cups of water (for preparation of boxed meals) to 90°F on a kitchen stove as quickly as possible. In this experiment, only one stove was used, and the three treatment factors were

C: diameter of pot (5.5,6.25 and 8.625 inches; coded 1, 2, 3)

D: burner size (small, large; coded 1, 2)

E: cover (no, yes; coded 1, 2).

   (a) [2pts]Using the factorial form of the block-treatment model similar to (10.8 .15), p. 325 , but with three treatment factors, test the hypotheses of no interactions between pairs of treatment factors, each test done at level 0.05. Do you see any significant interactions?

   (b) [2pts]Test the null hypothesis that the main effects of Factor C are all equal using $\alpha = 0.05$.

   (c) [3pts]Find 95% simultaneous confidence intervals for all pairwise differences of the main effects of Factor C. What do you conclude from these intervals?

   (d) [3pts, optional bonus]You will see that the width of the confidence intervals in (b) is 50.3. Suppose the experiment will be repeated in the future and what has been done will serve as a pilot study. Based on this experiment, the 90% confidence upper limit for σ^2 is 1200.336. Use this upper limit of σ^2 to calculate the necessary number of blocks for the 95% simultaneous confidence intervals for all pairwise comparisons of the main effects of C to have a width less than or equal to 40 when the block size is kept at 12. [Hint: If we write the response as Y_{hijk} where h denotes block, i, j, k denote factors C, D and E, respectively. The difference between the first two main effects is estimated by $\hat{Y}_{.1..}$-$\hat{Y}_{.2..}$. Think of the variance of this estimator.]

2. Consider the following incomplete block designs.

Design I		Design II		Design III	
Block	Treatments	Block	Treatments	Block	Treatments
1	12	1	123	1	126
2	13	2	456	2	345
3	14	3	789	3	268
4	23	4	147	4	457
5	24	5	258	5	168
6	34	6	369	6	357
		7	159	7	128
		8	267	8	347
		9	348		

   (a) [2pts]Design I is connected. (True or False)

   (b) [1pt]Draw a connectivity graph for Design I.

   (c) [2pts]Design II is connected. (True or False)

   (d) [1pt]Draw a connectivity graph for Design II.

(e) [2pts]Design III is a connected design. (True or False)

(f) [1pt]Draw a connectivity graph for Design III.

(g) [1pt] Which designs are balanced incomplete block designs (if any)?

3. Consider an experiment to compare 7 treatments in blocks of size 5, with an anticipated error variance of at most 30 squared units.

(a) [2pts]Verify that the following conditions cannot be satisfied with less than $b = 21$ blocks

$$vr = bk$$
$$r(k-1) = \lambda(v-1)$$

[Hint: For each $b \leq 20$, calculate r and λ according to the above formula. Show the $b = 21$ is the minimum that yields both an integer r and an integer λ.]

(b) [3pts]Assuming that a balanced incomplete block design exists with $b = 21$ blocks and $r = 15$, and the error variance is at most 30 squared units, calculate the width of 95% simultaneous confidence intervals of a pairwise comparison using Tukey's method.

(c) [3pts]Repeat part (b) using Dunnett's method for treatment versus control comparisons.

4. The rust experiment investigated the effect of temperature on the percentage of surface area of a metal sheet exhibiting rust after a given length of time exposed to certain weathering conditions. Five temperatures were examined in the experiment, but only three could be examined at any one time under identical experimental conditions. A balanced incomplete block design was used, formed from two cyclic designs with initial blocks $(1, 2, 3)$ and $(1, 2, 4)$. The data and design are shown in Table 11.28.

(a) [3pts]Use SAS to calculate a set of 95% simultaneous confidence intervals for pairwise comparisons among the temperatures in this experiment using Tukey's method. Provide your SAS code and the results.

(b) [3pts]Provide the least squares estimates of the percentage for each temperature, and plot them against the temperature.

(c) [2pts]Test the hypothesis that there is no linear trend in the percentage of rust as the temperature increases by providing the $p-$value and your conclusion using $\alpha = 0.01$ [Hint: First decide the contrast of coefficients for the linear trend. Then use the SAS command `contrast` or `estimate`].