29.

a. There are 26 letters, so allowing repeats there are $(26)(26) = (26)^2 = 676$ possible 2-letter domain names. Add in the 10 digits, and there are 36 characters available, so allowing repeats there are $(36)(36) = (36)^2 = 1296$ possible 2-character domain names.

b. By the same logic as part **a**, the answers are $(26)^3 = 17{,}576$ and $(36)^3 = 46{,}656$.

c. Continuing, $(26)^4 = 456{,}976$; $(36)^4 = 1{,}679{,}616$.

d. P(4-character sequence is already owned) = 1 – P(4-character sequence still available) = 1 – $97{,}786/(36)^4 = .942$.

31.

a. Use the Fundamental Counting Principle: $(9)(27) = 243$.

b. By the same reasoning, there are $(9)(27)(15) = 3645$ such sequences, so such a policy could be carried out for 3645 successive nights, or approximately 10 years, without repeating exactly the same program.

33.

a. Since there are 15 players and 9 positions, and order matters in a line-up (catcher, pitcher, shortstop, etc. are different positions), the number of possibilities is $P_{9,15} = (15)(14)\ldots(7)$ or $15!/(15–9)! = 1{,}816{,}214{,}440$.

b. For each of the starting line-ups in part (a), there are 9! possible batting orders. So, multiply the answer from (a) by 9! to get $(1{,}816{,}214{,}440)(362{,}880) = 659{,}067{,}881{,}472{,}000$.

c. Order still matters: There are $P_{3,5} = 60$ ways to choose three left-handers for the outfield and $P_{6,10} = 151{,}200$ ways to choose six right-handers for the other positions. The total number of possibilities is $= (60)(151{,}200) = 9{,}072{,}000$.

35.

a. Since there are 20 day-shift workers, the number of such samples is $\binom{20}{6} = 38{,}760$. With 45 workers total, there are $\binom{45}{6}$ total possible samples. So, the probability of randomly selecting all day-shift workers is $\dfrac{\binom{20}{6}}{\binom{45}{6}} = \dfrac{38{,}760}{8{,}145{,}060} = .0048$.

b. Following the analogy from **a**, P(all from the same shift) = P(all from day shift) + P(all from swing shift) + P(all from graveyard shift) $= \dfrac{\binom{20}{6}}{\binom{45}{6}} + \dfrac{\binom{15}{6}}{\binom{45}{6}} + \dfrac{\binom{10}{6}}{\binom{45}{6}} = .0048 + .0006 + .0000 = .0054$.

c. P(at least two shifts represented) = 1 – P(all from same shift) = 1 – .0054 = .9946.

d. There are several ways to approach this question. For example, let A_1 = "day shift is unrepresented," A_2 = "swing shift is unrepresented," and A_3 = "graveyard shift is unrepresented." Then we want $P(A_1 \cup A_2 \cup A_3)$.

$P(A_1)$ = P(day shift unrepresented) = P(all from swing/graveyard) $= \dfrac{\binom{25}{6}}{\binom{45}{6}}$,

since there are 15 + 10 = 25 total employees in the swing and graveyard shifts. Similarly, $P(A_2) = \dfrac{\binom{30}{6}}{\binom{45}{6}}$ and $P(A_3) = \dfrac{\binom{35}{6}}{\binom{45}{6}}$. Next, $P(A_1 \cap A_2)$ = P(all from graveyard) $= \dfrac{\binom{10}{6}}{\binom{45}{6}}$.

Similarly, $P(A_1 \cap A_3) = \dfrac{\binom{15}{6}}{\binom{45}{6}}$ and $P(A_2 \cap A_3) = \dfrac{\binom{20}{6}}{\binom{45}{6}}$. Finally, $P(A_1 \cap A_2 \cap A_3) = 0$, since at least one shift must be represented. Now, apply the addition rule for 3 events:

$$P(A_1 \cup A_2 \cup A_3) = \frac{\binom{25}{6}}{\binom{45}{6}} + \frac{\binom{30}{6}}{\binom{45}{6}} + \frac{\binom{35}{6}}{\binom{45}{6}} - \frac{\binom{10}{6}}{\binom{45}{6}} - \frac{\binom{15}{6}}{\binom{45}{6}} - \frac{\binom{20}{6}}{\binom{45}{6}} + 0 = .2885.$$

37.

a. By the Fundamental Counting Principle, with $n_1 = 3$, $n_2 = 4$, and $n_3 = 5$, there are (3)(4)(5) = 60 runs.

b. With $n_1 = 1$ (just one temperature), $n_2 = 2$, and $n_3 = 5$, there are (1)(2)(5) = 10 such runs.

c. For each of the 5 specific catalysts, there are (3)(4) = 12 pairings of temperature and pressure. Imagine we separate the 60 possible runs into those 5 sets of 12. The number of ways to select exactly one run from each of these 5 sets of 12 is $\binom{12}{1}^5 = 12^5$.

Since there are $\binom{60}{5}$ ways to select the 5 runs overall, the desired probability is $\dfrac{\binom{12}{1}^5}{\binom{60}{5}} = \dfrac{12^5}{\binom{60}{5}} = .0456$.

39.

a. We want to choose all of the 5 cordless, and 5 of the 10 others, to be among the first 10 serviced, so the desired probability is $\dfrac{\binom{5}{5}\binom{10}{5}}{\binom{15}{10}} = \dfrac{252}{3003} = .0839$.

b. Isolating one group, say the cordless phones, we want the other two groups (cellular and corded) represented in the last 5 serviced. The number of ways to choose all 5 cordless phones and 5 of the other phones in the first 10 selections is $\binom{5}{5}\binom{10}{5} = \binom{10}{5}$. However, we don't want two types to be eliminated in the first 10 selections, so we must subtract out the ways that either (all cordless and all cellular) or (all cordless and all corded) are selected among the first 10, which is $\binom{5}{5}\binom{5}{5} + \binom{5}{5}\binom{5}{5} = 2$.

So, the number of ways to have only cellular and corded phones represented in the last five selections is
$\binom{10}{5} - 2$. We have three types of phones, so the total number of ways to have exactly two types left over is $3 \cdot \left[\binom{10}{5} - 2\right]$, and the probability is $\dfrac{3 \cdot \left[\binom{10}{5} - 2\right]}{\binom{15}{5}} = \dfrac{3(250)}{3003} = .2498$.

c. We want to choose 2 of the 5 cordless, 2 of the 5 cellular, and 2 of the corded phones:
$\dfrac{\binom{5}{2}\binom{5}{2}\binom{5}{2}}{\binom{15}{6}} = \dfrac{1000}{5005} = .1998$.

41.

a. $(10)(10)(10)(10) = 10^4 = 10{,}000$. These are the strings 0000 through 9999.

b. Count the number of prohibited sequences. There are (i) 10 with all digits identical (0000, 1111, …, 9999); (ii) 14 with sequential digits (0123, 1234, 2345, 3456, 4567, 5678, 6789, and 7890, plus these same seven descending); (iii) 100 beginning with 19 (1900 through 1999). That's a total of 10 + 14 + 100 = 124 impermissible sequences, so there are a total of 10,000 – 124 = 9876 permissible sequences. The chance of randomly selecting one is just $\frac{9876}{10{,}000} = .9876$.

c. All PINs of the form 8xx1 are legitimate, so there are (10)(10) = 100 such PINs. With someone randomly selecting 3 such PINs, the chance of guessing the correct sequence is 3/100 = .03.

d. Of all the PINs of the form 1xx1, eleven is prohibited: 1111, and the ten of the form 19x1. That leaves 89 possibilities, so the chances of correctly guessing the PIN in 3 tries is 3/89 = .0337.

43. There are $\binom{52}{5} = 2{,}598{,}960$ five-card hands. The number of 10-high straights is $(4)(4)(4)(4)(4) = 4^5 = 1024$ (any of four 6s, any of four 7s, etc.). So, $P(\text{10 high straight}) = \frac{1024}{2{,}598{,}960} = .000394$. Next, there ten "types of straight: A2345, 23456, …, 910JQK, 10JQKA. So, $P(\text{straight}) = 10 \times \frac{1024}{2{,}598{,}960} = .00394$. Finally, there are only 40 straight flushes: each of the ten sequences above in each of the 4 suits makes (10)(4) = 40. So, $P(\text{straight flush}) = \frac{40}{2{,}598{,}960} = .00001539$.