22.

a. For a one-sided bound, we need $z_\alpha = z_{.05} = 1.645$; $\hat{p} = \frac{10}{143} = .07$; and $\tilde{p} = \frac{.07 + 1.645^2/(2\cdot 143)}{1 + 1.645^2/143} = .078$. The resulting 95% lower confidence bound for *p*, the true proportion of such artificial hip recipients that experience squeaking, is $.078 - \frac{1.645\sqrt{(.07)(.93)/143 + (1.645)^2/(4\cdot 143^2)}}{1 + (1.645)^2/143} = .078 - .036 = .042$. We are 95% confident that more than 4.2% of all such artificial hip recipients experience squeaking.

b. If we were to sample repeatedly, the calculation method in (a) is such that *p* will exceed the calculated lower confidence bound for 95% of all possible random samples of $n = 143$ individuals who received ceramic hips. (We hope that <u>our</u> sample is among that 95%!)

23.

a. With such a large sample size, we can use the "simplified" CI formula (7.11). With $\hat{p} = .25$, $n = 2003$, and $z_{\alpha/2} = z_{.005} = 2.576$, the 99% confidence interval for *p* is

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = .25 \pm 2.576\sqrt{\frac{(.25)(.75)}{2003}} = .25 \pm .025 = (.225, .275).$$

b. Using the "simplified" formula for sample size and $\hat{p} = \hat{q} = .5$,

$$n = \frac{4z^2\hat{p}\hat{q}}{w^2} = \frac{4(2.576)^2(.5)(.5)}{(.05)^2} = 2654.31$$

So, a sample of size at least 2655 is required. (We use $\hat{p} = \hat{q} = .5$ here, rather than the values from the sample data, so that our CI has the desired width irrespective of what the true value of *p* might be. See the textbook discussion toward the end of Section 7.2.)

30.

a. $t_{.025,10} = 2.228$

b. $t_{.025,15} = 2.131$

c. $t_{.005,15} = 2.947$

d. $t_{.005,4} = 4.604$

e. $t_{.01,24} = 2.492$

f. $t_{.005,37} \approx 2.712$

34. $n = 14$, $\bar{x} = 8.48$, $s = .79$; $t_{.05,13} = 1.771$

a. A 95% lower confidence bound: $8.48 - 1.771\left(\frac{.79}{\sqrt{14}}\right) = 8.48 - .37 = 8.11$. With 95% confidence, the value of the true mean proportional limit stress of all such joints is greater than 8.11 MPa. We must assume that the sample observations were taken from a normally distributed population.

b. A 95% lower prediction bound: $8.48 - 1.771(.79)\sqrt{1 + \frac{1}{14}} = 8.48 - 1.45 = 7.03$. If this bound is calculated for sample after sample, in the long run 95% of these bounds will provide a lower bound for the corresponding future values of the proportional limit stress of a single joint of this type.

44. $n-1=8$, $\chi^2_{.025,8}=17.534$, $\chi^2_{.975,8}=2.180$, so the 95% interval for σ^2 is $\left(\frac{8(7.90)}{17.534},\frac{8(7.90)}{2.180}\right)=(3.60,28.98)$. The 95% interval for σ is $\left(\sqrt{3.60},\sqrt{28.98}\right)=(1.90,5.38)$.

46.

a. Using a normal probability plot, we ascertain that it is plausible that this sample was taken from a normal population distribution.

b. With $s=1.579$, $n=15$, and $\chi^2_{.95,14}=6.571$, the 95% upper confidence bound for σ is

$$\sqrt{\frac{14(1.579)^2}{6.571}}=2.305.$$

50. $\overline{x}=$ the midpoint of the interval $=\frac{229.764+233.504}{2}=231.634$. To find s we use width $=2t_{.025,4}\left(\frac{s}{\sqrt{n}}\right)$, and solve for s. Here, $n=5$, $t_{.025,4}=2.776$, and width = upper limit – lower limit = 3.74.

$3.74=2(2776)\frac{s}{\sqrt{5}}\Rightarrow s=\frac{\sqrt{5}(3.74)}{2(2.776)}=1.5063$. So for a 99% CI, $t_{.005,4}=4.604$, and the interval is

$$231.634\pm4.604\frac{1.5063}{\sqrt{5}}=213.634\pm3.101=(228.533,234.735).$$

3. In this formulation, H_0 states the welds do not conform to specification. This assertion will not be rejected unless there is strong evidence to the contrary. Thus the burden of proof is on those who wish to assert that the specification is satisfied. Using H_a: $\mu<100$ results in the welds being believed in conformance unless proved otherwise, so the burden of proof is on the non-conformance claim.

4. When the alternative is H_a: $\mu<5$, the formulation is such that the water is believed unsafe until proved otherwise. A type I error involved deciding that the water is safe (rejecting H_0) when it isn'*t* (H_0 is true). This is a very serious error, so a test which ensures that this error is highly unlikely is desirable. A type II error involves judging the water unsafe when it is actually safe. Though a serious error, this is less so than the type I error. It is generally desirable to formulate so that the type I error is more serious, so that the probability of this error can be explicitly controlled. Using H_a: $\mu>5$, the type II error (now stating that the water is safe when it isn'*t*) is the more serious of the two errors.

18.

a. $\dfrac{72.3-75}{1.8}=-1.5$ so 72.3 is 1.5 SDs (of $\overline{x}$) below 75.

b. H_0 is rejected if $z \le -2.33$; since $z = -1.5$ is not ≤ -2.33, don't reject H_0.

c. α = area under standard normal curve below –2.88 = Φ(–2.88) = .0020.

d. $\Phi\left(-2.88+\dfrac{75-70}{9/5}\right)=\Phi(-.1)=.4602$ so $\beta(70)=.5398$.

e. $n=\left[\dfrac{9(2.88+2.33)}{75-70}\right]^2=87.95$, so use $n=88$.

f. Zero. By definition, a type I error can only occur when H_0 is true, but $\mu = 76$ means that H_0 is actually false.

29.

a. The hypotheses are H_0: $\mu = 200$ versus H_a: $\mu > 200$. H_0 will be rejected at level $\alpha = .05$ if $t \ge t_{.05,12-1} = t_{.05,11} = 1.796$. With the data provided, $t=\dfrac{\overline{x}-\mu_0}{s/\sqrt{n}}=\dfrac{249.7-200}{145.1/\sqrt{12}}=1.19$. Since 1.19 < 1.796, H_0 is not rejected at the $\alpha = .05$ level. We have insufficient evidence to conclude that the true average repair time exceeds 200 minutes.

b. With $d=\dfrac{|\mu_0-\mu|}{\sigma}=\dfrac{|200-300|}{150}=0.67$, df = 11, and $\alpha = .05$, software calculates power ≈ .70, so $\beta(300) \approx .30$.

34. $n = 12$, $\overline{x}=98.375$, $s = 6.1095$

a.

1 Parameter of Interest: μ = true average reading of this type of radon detector when exposed to 100 pCi/L of radon.

2 Null Hypothesis: H_0: $\mu = 100$

3 Alternative Hypothesis: H_a: $\mu \ne 100$

4 $t=\dfrac{\overline{x}-\mu_o}{s/\sqrt{n}}=\dfrac{\overline{x}-100}{s/\sqrt{n}}$

5 $t \le -2.201$ or $t \ge 2.201$

6 $t=\dfrac{98.375-100}{6.1095/\sqrt{12}}=-.9213$

7 Fail to reject H_0. The data does not indicate that these readings differ significantly from 100.

b. $\sigma = 7.5$, $\beta = 0.10$, $d = 0.67$. From table A.17, df ≈ 29, thus $n \approx 30$.