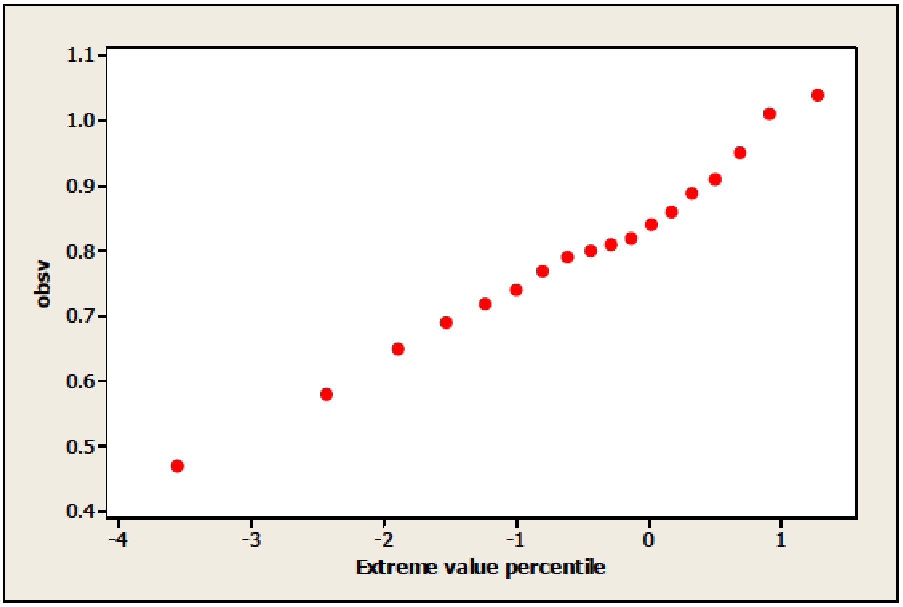
90. The Weibull plot uses ln(observations) and the extreme value percentiles of the p_i values given; i.e., $\eta(p) = \ln[-\ln(1-p)]$. The accompanying probability plot appears sufficiently straight to lead us to agree with the argument that the distribution of fracture toughness in concrete specimens could well be modeled by a Weibull distribution.



100.

a. Clearly $f(x) \geq 0$. Now check that the function integrates to 1:

$$\int_0^\infty \frac{32}{(x+4)^3}dx = \int_0^\infty 32(x+4)^{-3}\,dx = -\frac{16}{(x+4)^2}\Bigg|_0^\infty = 0 - -\frac{16}{(0+4)^2} = 1.$$

b. For $x \leq 0$, $F(x) = 0$. For $x > 0$,

$$F(x) = \int_{-\infty}^x f(y)dy = \int_0^x \frac{32}{(y+4)^3}dy = -\frac{1}{2}\cdot\frac{32}{(y+4)^2}\Bigg]_0^x = 1 - \frac{16}{(x+4)^2}.$$

c. $P(2 \leq X \leq 5) = F(5) - F(2) = 1 - \frac{16}{81} - \left(1 - \frac{16}{36}\right) = .247$.

d. $E(X) = \int_{-\infty}^\infty x\cdot f(x)dx = \int_{-\infty}^\infty x\cdot\frac{32}{(x+4)^3}dx = \int_0^\infty (x+4-4)\cdot\frac{32}{(x+4)^3}dx$

$$= \int_0^\infty \frac{32}{(x+4)^2}dx - 4\int_0^\infty \frac{32}{(x+4)^3}dx = 8 - 4 = 4 \text{ years.}$$

e. $E\left(\frac{100}{X+4}\right) = \int_0^\infty \frac{100}{x+4}\cdot\frac{32}{(x+4)^3}dx = 3200\int_0^\infty \frac{1}{(x+4)^4}dx = \frac{3200}{(3)(64)} = 16.67$.

119.

a. $Y = -\ln(X) \Rightarrow x = e^{-y} = k(y)$, so $k'(y) = -e^{-y}$. Thus since $f(x) = 1$, $g(y) = 1 \cdot |-e^{-y}| = e^{-y}$ for $0 < y < \infty$. Y has an exponential distribution with parameter $\lambda = 1$.

b. $y = \sigma Z + \mu \Rightarrow z = k(y) = \dfrac{y-\mu}{\sigma}$ and $k'(y) = \dfrac{1}{\sigma}$, from which the result follows easily.

c. $y = h(x) = cx \Rightarrow x = k(y) = \dfrac{y}{c}$ and $k'(y) = \dfrac{1}{c}$, from which the result follows easily.

46.

a. The sampling distribution of $\bar{X}$ is centered at $E(\bar{X}) = \mu = 12$ cm, and the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \dfrac{\sigma_{\bar{X}}}{\sqrt{n}} = \dfrac{.04}{\sqrt{16}} = .01$ cm.

b. With $n = 64$, the sampling distribution of $\bar{X}$ is still centered at $E(\bar{X}) = \mu = 12$ cm, but the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \dfrac{\sigma_{\bar{X}}}{\sqrt{n}} = \dfrac{.04}{\sqrt{64}} = .005$ cm.

c. $\bar{X}$ is more likely to be within .01 cm of the mean (12 cm) with the second, larger, sample. This is due to the decreased variability of $\bar{X}$ that comes with a larger sample size.

49.

a. 11 P.M. – 6:50 P.M. = 250 minutes. With $T_o = X_1 + \ldots + X_{40}$ = total grading time, $\mu_{T_o} = n\mu = (40)(6) = 240$ and $\sigma_{T_o} = \sigma \cdot \sqrt{n} = 37.95$, so $P(T_o \le 250) \approx$

$$P\left(Z \le \frac{250-240}{37.95}\right) = P(Z \le .26) = .6026.$$

b. The sports report begins 260 minutes after he begins grading papers.

$$P(T_0 > 260) = P\left(Z > \frac{260-240}{37.95}\right) = P(Z > .53) = .2981.$$

50.

a. $P(9{,}900 \le \bar{X} \le 10{,}200) \approx P\left(\dfrac{9{,}900-10{,}000}{500/\sqrt{40}} \le Z \le \dfrac{10{,}200-10{,}000}{500/\sqrt{40}}\right)$

$= P(-1.26 \le Z \le 2.53) = \Phi(2.53) - \Phi(-1.26) = .9943 - .1038 = .8905.$

b. According to the guideline given in Section 5.4, n should be greater than 30 in order to apply the CLT, thus using the same procedure for $n = 15$ as was used for $n = 40$ would not be appropriate.

51. Individual times are given by $X \sim N(10, 2)$. For day 1, $n = 5$, and so

$$P(\overline{X} \le 11) = P\left(Z \le \frac{11-10}{2/\sqrt{5}}\right) = P(Z \le 1.12) = .8686 .$$

For day 2, $n = 6$, and so

$$P(\overline{X} \le 11) = P(\overline{X} \le 11) = P\left(Z \le \frac{11-10}{2/\sqrt{6}}\right) = P(Z \le 1.22) = .8888 .$$

Finally, assuming the results of the two days are independent (which seems reasonable), the probability the sample average is at most 11 min on both days is (.8686)(.8888) = .7720.

52. We have $X \sim N(10,1)$, $n = 4$, $\mu_{T_o} = n\mu = (4)(10) = 40$ and $\sigma_{T_o} = \sigma\sqrt{n} = 2$. Hence, $T_o \sim N(40, 2)$. We desire the 95$^{\text{th}}$ percentile of T_o: 40 + (1.645)(2) = 43.29 hours.