2. $f(x) = \frac{1}{10}$ for $-5 \le x \le 5$ and $= 0$ otherwise

a. $P(X < 0) = \int_{-5}^{0} \frac{1}{10} dx = .5$.

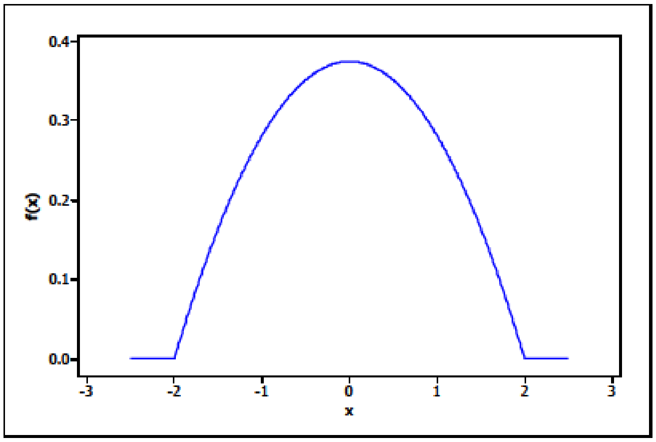
b. $P(-2.5 < X < 2.5) = \int_{-2.5}^{2.5} \frac{1}{10} dx = .5$.

c. $P(-2 \le X \le 3) = \int_{-2}^{3} \frac{1}{10} dx = .5$.

d. $P(k < X < k + 4) = \int_{k}^{k+4} \frac{1}{10} dx = \frac{1}{10} x \Big]_{k}^{k+4} = \frac{1}{10}[(k+4) - k] = .4$.

3.

a.



b. $P(X > 0) = \int_{0}^{2} .09375(4 - x^2) dx = .09375\left(4x - \frac{x^3}{3}\right)\Big]_{0}^{2} = .5$.

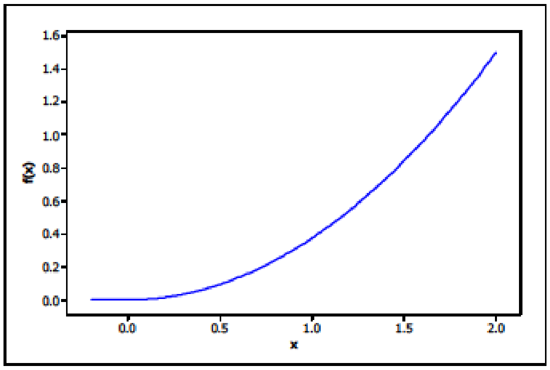
This matches the symmetry of the pdf about $x = 0$.

c. $P(-1 < X < 1) = \int_{-1}^{1} .09375(4 - x^2) dx = .6875$.

d. $P(X < -.5$ or $X > .5) = 1 - P(-.5 \le X \le .5) = 1 - \int_{-.5}^{.5} .09375(4 - x^2) dx = 1 - .3672 = .6328$.

5.

a. $1 = \int_{-\infty}^{\infty} f(x)dx = \int_0^2 kx^2 dx = \frac{kx^3}{3}\Big]_0^2 = \frac{8k}{3} \Rightarrow k = \frac{3}{8}$.



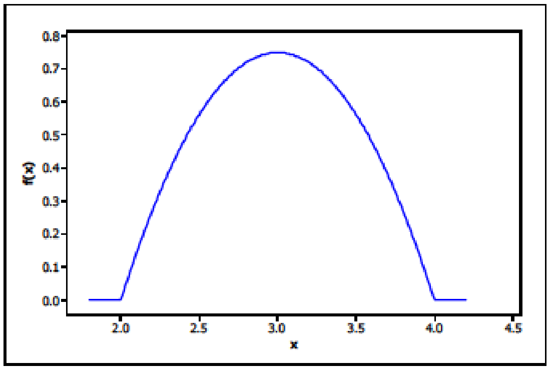
b. $P(0 \le X \le 1) = \int_0^1 \frac{3}{8}x^2 dx = \frac{1}{8}x^3\Big]_0^1 = \frac{1}{8} = .125$.

c. $P(1 \le X \le 1.5) = \int_1^{1.5} \frac{3}{8}x^2 dx = \frac{1}{8}x^3\Big]_1^{1.5} = \frac{1}{8}\left(\frac{3}{2}\right)^3 - \frac{1}{8}(1)^3 = \frac{19}{64} = .296875$.

d. $P(X \ge 1.5) = 1 - \int_{1.5}^2 \frac{3}{8}x^2 dx = \frac{1}{8}x^3\Big]_{1.5}^2 = \frac{1}{8}(2)^3 - \frac{1}{8}(1.5)^3 = .578125$.

6.

a.



b. $1 = \int_2^4 k[1-(x-3)^2]dx = \int_{-1}^1 k[1-u^2]du = \cdots = \frac{4k}{3} \Rightarrow k = \frac{3}{4}$.

c. $P(X > 3) = \int_3^4 \frac{3}{4}[1-(x-3)^2]dx = .5$. This matches the symmetry of the pdf about $x = 3$.

d. $P\left(\frac{11}{4} \le X \le \frac{13}{4}\right) = \int_{11/4}^{13/4} \frac{3}{4}[1-(x-3)^2]dx = \frac{3}{4}\int_{-1/4}^{1/4}[1-u^2]du = \frac{47}{128} \approx .367$.

e. $P(|X-3| > .5) = 1 - P(|X-3| \le .5) = 1 - P(2.5 \le X \le 3.5) =$
$1 - \int_{-.5}^{.5} \frac{3}{4}[1-u^2]du = \cdots = 1 - .6875 = .3125$.

11.

a. $P(X \le 1) = F(1) = \frac{1^2}{4} = .25$.

b. $P(.5 \le X \le 1) = F(1) - F(.5) = \frac{1^2}{4} - \frac{.5^2}{4} = .1875.$

c. $P(X > 1.5) = 1 - P(X \le 1.5) = 1 - F(1.5) = 1 - \frac{1.5^2}{4} = .4375.$

d. $.5 = F(\tilde{\mu}) = \frac{\tilde{\mu}^2}{4} \Rightarrow \tilde{\mu}^2 = 2 \Rightarrow \tilde{\mu} = \sqrt{2} \approx 1.414$.

e. $f(x) = F'(x) = \frac{x}{2}$ for $0 \le x < 2$, and $= 0$ otherwise.

f. $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx = \int_0^2 x \cdot \frac{x}{2}dx = \frac{1}{2}\int_0^2 x^2 dx = \frac{x^3}{6}\Big]_0^2 = \frac{8}{6} \approx 1.333$.

g. $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_0^2 x^2 \frac{x}{2}dx = \frac{1}{2}\int_0^2 x^3 dx = \frac{x^4}{8}\Big]_0^2 = 2$, so $V(X) = E(X^2) - [E(X)]^2 =$

$2 - \left(\frac{8}{6}\right)^2 = \frac{8}{36} \approx .222$, and $\sigma_X = \sqrt{.222} = .471.$

h. From **g**, $E(X^2) = 2.$

12.

a. $P(X < 0) = F(0) = .5.$

b. $P(-1 \le X \le 1) = F(1) - F(-1) = .6875.$

c. $P(X > .5) = 1 - P(X \le .5) = 1 - F(.5) = 1 - .6836 = .3164.$

d. $f(x) = F'(x) = \frac{d}{dx}\left(\frac{1}{2} + \frac{3}{32}\left(4x - \frac{x^3}{3}\right)\right) = 0 + \frac{3}{32}\left(4 - \frac{3x^2}{3}\right) = .09375\left(4 - x^2\right)$.

e. By definition, $F(\tilde{\mu}) = .5$. $F(0) = .5$ from **a** above, which is as desired.

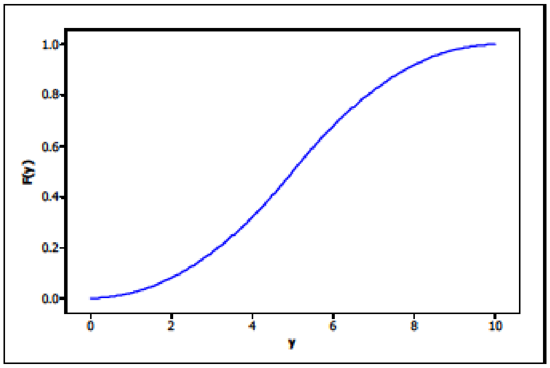
20.

a. For $0 \le y < 5$, $F(y) = \int_0^y \frac{u}{25} du = \frac{y^2}{50}$; for $5 \le y \le 10$,

$$F(y) = \int_0^y f(u)du = \int_0^5 f(u)du + \int_5^y f(u)du = \frac{5^2}{50} + \int_5^y \left(\frac{2}{5} - \frac{u}{25}\right)du = \cdots = -\frac{y^2}{50} + \frac{2}{5}y - 1$$

So, the complete cdf of Y is

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{y^2}{50} & 0 \le y < 5 \\ -\frac{y^2}{50} + \frac{2}{5}y - 1 & 5 \le y < 10 \\ 1 & y \ge 10 \end{cases}$$



b. In general, set $F(y) = p$ and solve for y.

For $0 < p < .5$, $p = F(y) = \frac{y^2}{50} \Rightarrow \eta(p) = y = \sqrt{50p}$; for $.5 \le p < 1$,

$$p = -\frac{y^2}{50} + \frac{2}{5}y - 1 \Rightarrow \eta(p) = y = 10 - 5\sqrt{2(1-p)}.$$

c. $E(Y) = 5$ by straightforward integration, or by the symmetry of $f(y)$ about $y = 5$.

Similarly, by symmetry $V(Y) = \int_0^{10} (y-5)^2 f(y)\,dy = 2\int_0^5 (y-5)^2 f(y)\,dy = 2\int_0^5 (y-5)^2 \frac{y^2}{50} dy = \ldots = \frac{50}{12} =$ 4.1667. For the waiting time X for a single bus, $E(X) = 2.5$ and $V(X) = \frac{25}{12}$; not coincidentally, the mean and variance of Y are exactly twice that of X.

21. $E(\text{area}) = E(\pi R^2) = \int_{-\infty}^{\infty} \pi r^2 f(r)dr = \int_9^{11} \pi r^2 \frac{3}{4}\left(1-(10-r)^2\right)dr = \cdots = \frac{501}{5}\pi = 314.79 \text{ m}^2$.

22.

a. For $1 \le x \le 2$, $F(x) = \int_1^x 2\left(1 - \frac{1}{y^2}\right)dy = 2\left(y + \frac{1}{y}\right)\Big]_1^x = 2\left(x + \frac{1}{x}\right) - 4$, so the cdf is

$$F(x) = \begin{cases} 0 & x < 1 \\ 2\left(x + \frac{1}{x}\right) - 4 & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

b. Set $F(x) = p$ and solve for x: $2\left(x + \frac{1}{x}\right) - 4 = p \Rightarrow 2x^2 - (p+4)x + 2 = 0 \Rightarrow$

$\eta(p) = x = \frac{(p+4) + \sqrt{(p+4)^2 - 4(2)(2)}}{2(2)} = \frac{p + 4 + \sqrt{p^2 + 8p}}{4}$. (The other root of the quadratic gives solutions outside the interval [1, 2].) To find the median $\tilde{\mu}$, set $p = .5$: $\tilde{\mu} = \eta(.5) = \ldots = 1.640$.

c. $E(X) = \int_1^2 x \cdot 2\left(1 - \frac{1}{x^2}\right)dx = 2\int_1^2 \left(x - \frac{1}{x}\right)dx = 2\left(\frac{x^2}{2} - \ln(x)\right)\Big]_1^2 = 1.614$. Similarly,

$E(X^2) = 2\int_1^2 \left(x^2 - 1\right)dx = 2\left(\frac{x^3}{3} - x\right)\Big]_1^2 = \frac{8}{3} \Rightarrow V(X) = .0626$.

d. The amount left is given by $h(x) = \max(1.5 - x, 0)$, so

$E(h(X)) = \int_1^2 \max(1.5 - x, 0) f(x)dx = 2\int_1^{1.5} (1.5 - x)\left(1 - \frac{1}{x^2}\right)dx = .061$.

35.

a. $P(X \ge 10) = P(Z \ge .43) = 1 - \Phi(.43) = 1 - .6664 = .3336$.
Since X is continuous, $P(X > 10) = P(X \ge 10) = .3336$.

b. $P(X > 20) = P(Z > 4) \approx 0$.

c. $P(5 \le X \le 10) = P(-1.36 \le Z \le .43) = \Phi(.43) - \Phi(-1.36) = .6664 - .0869 = .5795$.

d. $P(8.8 - c \le X \le 8.8 + c) = .98$, so $8.8 - c$ and $8.8 + c$ are at the 1st and the 99th percentile of the given distribution, respectively. The 99th percentile of the standard normal distribution satisfies $\Phi(z) = .99$, which corresponds to $z = 2.33$.
So, $8.8 + c = \mu + 2.33\sigma = 8.8 + 2.33(2.8) \Rightarrow c = 2.33(2.8) = 6.524$.

e. From **a**, $P(X > 10) = .3336$, so $P(X \le 10) = 1 - .3336 = .6664$. For four independent selections,
P(at least one diameter exceeds 10) = 1 – P(none of the four exceeds 10) =
1 – P(first doesn't $\cap$ … fourth doesn't) = 1 – (.6664)(.6664)(.6664)(.6664) by independence =
$1 - (.6664)^4 = .8028$.

36.

a. $P(X < 1500) = P(Z < 3) = \Phi(3) = .9987$; $P(X \geq 1000) = P(Z \geq -.33) = 1 - \Phi(-.33) = 1 - .3707 = .6293$.

b. $P(1000 < X < 1500) = P(-.33 < Z < 3) = \Phi(3) - \Phi(-.33) = .9987 - .3707 = .6280$

c. From the table, $\Phi(z) = .02 \Rightarrow z = -2.05 \Rightarrow x = 1050 - 2.05(150) = 742.5$ μm. The smallest 2% of droplets are those smaller than 742.5 μm in size.

d. P(at least one droplet in 5 that exceeds 1500 μm) = 1 – P(all 5 are less than 1500 μm) = $1 - (.9987)^5 = 1 - .9935 = .0065$.

38. Let X denote the diameter of a randomly selected cork made by the first machine, and let Y be defined analogously for the second machine.
$P(2.9 \leq X \leq 3.1) = P(-1.00 \leq Z \leq 1.00) = .6826$, while
$P(2.9 \leq Y \leq 3.1) = P(-7.00 \leq Z \leq 3.00) = .9987$. So, the second machine wins handily.

50. We use a normal approximation to the binomial distribution: Let X denote the number of people in the sample of 1000 who can taste the difference, so $X \sim \text{Bin}(1000, .03)$. Because $\mu = np = 1000(.03) = 30$ and $\sigma = \sqrt{np(1-p)} = 5.394$, X is approximately $N(30, 5.394)$.

a. Using a continuity correction, $P(X \geq 40) = 1 - P(X \leq 39) = 1 - P\left(Z \leq \frac{39.5 - 30}{5.394}\right) =$

$1 - P(Z \leq 1.76) = 1 - \Phi(1.76) = 1 - .9608 = .0392$.

b. 5% of 1000 is 50, and $P(X \leq 50) = P\left(Z \leq \frac{50.5 - 30}{5.394}\right) = \Phi(3.80) \approx 1$.