68.

a. There are 20 items total, 12 of which are "successes" (two slots). Among these 20 items, 6 have been randomly selected to be put under the shelf. So, the random variable X is hypergeometric, with $N = 20$, $M = 12$, and $n = 6$.

b. $$P(X=2)=\frac{\binom{12}{2}\binom{20-12}{6-2}}{\binom{20}{6}}=\frac{\binom{12}{2}\binom{8}{4}}{\binom{20}{6}}=\frac{(66)(70)}{(38760)}=.1192.$$

$$P(X\le 2)=P(X=0)+P(X=1)+P(X=2)=\frac{\binom{12}{0}\binom{8}{6}}{\binom{20}{6}}+\frac{\binom{12}{1}\binom{8}{5}}{\binom{20}{6}}+.1192=$$

$.0007 + .0174 + .1192 = .1373.$
$P(X \ge 2) = 1 - P(X \le 1) = 1 - [P(X = 0) + P(X = 1)] = 1 - [.0007 + .0174] = .9819.$

c. $$E(X)=n\cdot\frac{M}{N}=6\cdot\frac{12}{20}=6\cdot(.6)=3.6;\ V(X)=\left(\frac{20-6}{20-1}\right)\cdot 6(.6)(1-.6)=1.061;\ \sigma=1.030.$$

69. According to the problem description, X is hypergeometric with $n = 6$, $N = 12$, and $M = 7$.

a. $$P(X=5)=\frac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}}=\frac{105}{924}=.114\,.$$

b. $$P(X\le 4)=1-P(X>4)=1-[P(X=5)+P(X=6)]=1-\left[\frac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}}+\frac{\binom{7}{6}\binom{5}{0}}{\binom{12}{6}}\right]=$$

$1 - [.114 + .007] = 1 - .121 = .879.$

c. $$E(X)=n\cdot\frac{M}{N}=6\cdot\frac{7}{12}=3.5;\ V(X)=\left(\frac{12-6}{12-1}\right)6\left(\frac{7}{12}\right)\left(1-\frac{7}{12}\right)=0.795;\ \sigma=0.892.\ \text{So,}$$
$P(X > \mu + \sigma) = P(X > 3.5 + 0.892) = P(X > 4.392) = P(X = 5 \text{ or } 6) = .121$ (from part **b**).

d. We can approximate the hypergeometric distribution with the binomial if the population size and the number of successes are large. Here, $n = 15$ and $M/N = 40/400 = .1$, so $h(x;15, 40, 400) \approx b(x;15, .10)$. Using this approximation, $P(X \le 5) \approx B(5; 15, .10) = .998$ from the binomial tables. (This agrees with the exact answer to 3 decimal places.)

75.

a. With S = a female child and F = a male child, let X = the number of F's before the 2[nd] S. Then
$P(X = x) = nb(x; 2, .5) = \binom{x+2-1}{2-1}(.5)^2(1-.5)^x = (x+1)(.5)^{x+2}$.

b. P(exactly 4 children) = P(exactly 2 males = $P(X = 2) = nb(2; 2, .5) = (2 + 1)(.5)^4 = .188$.

c. P(at most 4 children) = $P(X \le 2) = \sum_{x=0}^{2} nb(x;2,.5) = .25 + .25 + .188 = .688$.

d. $E(X) = \frac{r(1-p)}{p} = \frac{2(1-.5)}{.5} = 2$, so the expected number of children is equal to
$E(X + 2) = E(X) + 2 = 4$.

76. This question relates to the negative binomial distribution, but we can't use it directly (X, as it's defined, doesn't fit the negative binomial description). Instead, let's reason it out.
Clearly, X can't be 0, 1, or 2.
$P(X = 3) = P(BBB \text{ or } GGG) = (.5)^3 + (.5)^3 = .25$.
$P(X = 4) = P(BBGB \text{ or } BGBB \text{ or } GBBB \text{ or } GGBG \text{ or } GBGG \text{ or } BGGG) = 6(.5)^4 = .375$.
$P(X = 5)$ look scary until we realize that 5 is the maximum possible value of X!
(If you have 5 kids, you must have at least 3 of one gender.) So, $P(X = 5) = 1 - .25 - .375 = .375$, and that completes the pmf of X.

x	3	4	5
$p(x)$	.25	.375	.375

86.

a. $P(X = 4) = \frac{e^{-5}5^4}{4!} = .175$.

b. $P(X \ge 4) = 1 - P(X \le 3) = 1 - F(3; 5) = 1 - .265 = .735$.

c. Arrivals occur at the rate of 5 per hour, so for a 45-minute period the mean is $\mu = (5)(.75) = 3.75$, which is the expected number of arrivals in a 45-minute period.

87.

a. For a two hour period the parameter of the distribution is $\mu = \alpha t = (4)(2) = 8$,
so $P(X = 10) = \frac{e^{-8}8^{10}}{10!} = .099$.

b. For a 30-minute period, $\alpha t = (4)(.5) = 2$, so $P(X = 0) = \frac{e^{-2}2^0}{0!} = .135$.

c. The expected value is simply $E(X) = \alpha t = 2$.