13.

a. $P(X \le 3) = p(0) + p(1) + p(2) + p(3) = .10+.15+.20+.25 = .70$.

b. $P(X < 3) = P(X \le 2) = p(0) + p(1) + p(2) = .45$.

c. $P(X \ge 3) = p(3) + p(4) + p(5) + p(6) = .55$.

d. $P(2 \le X \le 5) = p(2) + p(3) + p(4) + p(5) = .71$.

e. The number of lines <u>not</u> in use is $6 - X$, and $P(2 \le 6 - X \le 4) = P(-4 \le -X \le -2) =$ $P(2 \le X \le 4) = p(2) + p(3) + p(4) = .65$.

f. $P(6 - X \ge 4) = P(X \le 2) = .10 + .15 + .20 = .45$.

14.

a. As the hint indicates, the sum of the probabilities must equal 1. Applied here, we get $\sum_{y=1}^{5} p(y) = k[1 + 2 + 3 + 4 + 5] = 15k = 1 \Rightarrow k = \frac{1}{15}$. In other words, the probabilities of the five y-values are $\frac{1}{15}, \frac{2}{15}, \frac{3}{15}, \frac{4}{15}, \frac{5}{15}$.

b. $P(Y \le 3) = P(Y = 1, 2, 3) = \frac{1}{15} + \frac{2}{15} + \frac{3}{15} = \frac{6}{15} = .4$.

c. $P(2 \le Y \le 4) = P(Y = 2, 3, 4) = \frac{2}{15} + \frac{3}{15} + \frac{4}{15} = \frac{9}{15} = .6$.

d. Do the probabilities total 1? Let's check: $\sum_{y=1}^{5} \left(\frac{y^2}{50} \right) = \frac{1}{50}[1 + 4 + 9 + 16 + 25] = \frac{55}{50} \ne 1$. No, that formula cannot be a pmf.

17.

a. $p(2) = P(Y = 2) = P(\text{first 2 batteries are acceptable}) = P(AA) = (.9)(.9) = .81$.

b. $p(3) = P(Y = 3) = P(UAA \text{ or } AUA) = (.1)(.9)^2 + (.1)(.9)^2 = 2[(.1)(.9)^2] = .162$.

c. The fifth battery must be an A, and exactly one of the first four must also be an A. Thus, $p(5) = P(AUUUA \text{ or } UAUUA \text{ or } UUAUA \text{ or } UUUAA) = 4[(.1)^3(.9)^2] = .00324$.

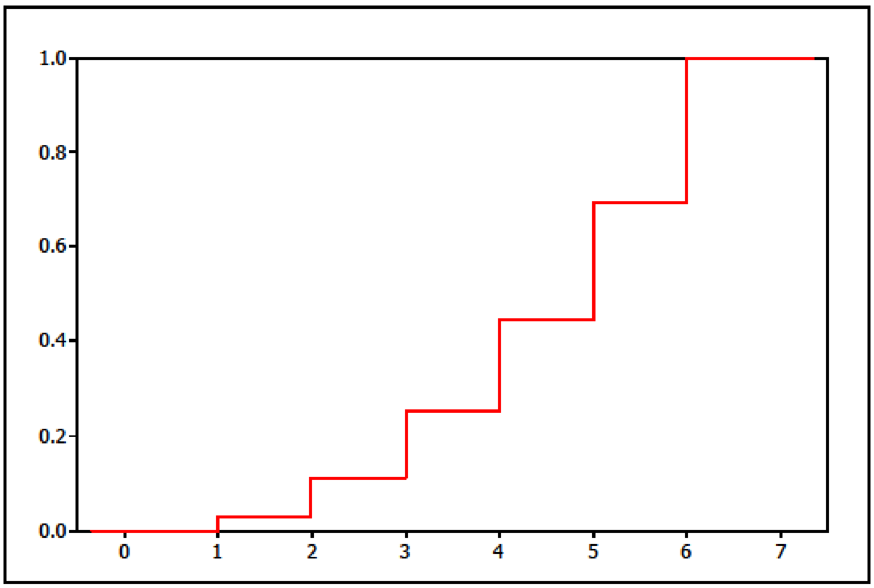
d. $p(y) = P(\text{the } y^{\text{th}} \text{ is an } A \text{ and so is exactly one of the first } y - 1) = (y - 1)(.1)^{y-2}(.9)^2$, for $y = 2, 3, 4, 5, \ldots$.

18.

a. $p(1) = P(M = 1) = P(\{(1,1)\}) = \frac{1}{36}$; $p(2) = P(M = 2) = P(\{(1,2)(2,1)(2,2)\}) = \frac{3}{36}$; $p(3) = P(M = 3) = P(\{(1,3)(2,3)(3,1)(3,2)(3,3)\}) = \frac{5}{36}$. Continuing the pattern, $p(4) = \frac{7}{36}$, $p(5) = \frac{9}{36}$, and $p(6) = \frac{11}{36}$.

b. Using the values in **a**,

$$F(m) = \begin{cases} 0 & m < 1 \\ \frac{1}{36} & 1 \le m < 2 \\ \frac{4}{36} & 2 \le m < 3 \\ \frac{9}{36} & 3 \le m < 4 \\ \frac{16}{36} & 4 \le m < 5 \\ \frac{25}{36} & 5 \le m < 6 \\ 1 & m \ge 6 \end{cases}$$



23.

a. $p(2) = P(X = 2) = F(3) - F(2) = .39 - .19 = .20.$

b. $P(X > 3) = 1 - P(X \le 3) = 1 - F(3) = 1 - .67 = .33.$

c. $P(2 \le X \le 5) = F(5) - F(2–1) = F(5) - F(1) = .92 - .19 = .78.$

d. $P(2 < X < 5) = P(2 < X \le 4) = F(4) - F(2) = .92 - .39 = .53.$

24.

a. Possible X values are those values at which $F(x)$ jumps, and the probability of any particular value is the size of the jump at that value. Thus we have:

x	1	3	4	6	12
$p(x)$	.30	.10	.05	.15	.40

b. $P(3 \le X \le 6) = F(6) - F(3-) = .60 - .30 = .30$; $P(4 \le X) = 1 - P(X < 4) = 1 - F(4-) = 1 - .40 = .60$.

32.

a. $E(X) = (13.5)(.2) + (15.9)(.5) + (19.1)(.3) = 16.38$; $E(X^2) = (13.5)^2(.2) + (15.9)^2(.5) + (19.1)^2(.3) = 272.298$. Put these together, and $V(X) = E(X^2) - [E(X)]^2 = 272.298 - (16.38)^2 = 3.9936$.

b. Use the linearity/rescaling property: $E(25X - 8.5) = 25\mu - 8.5 = 25(16.38) - 8.5 = \$401$. Alternatively, you can figure out the price for each of the three freezer types and take the weighted average.

c. Use the linearity/rescaling property again: $V(25X - 8.5) = 25^2\sigma^2 = 25^2(3.9936) = 2496$. (The 25 gets squared because variance is itself a square quantity.)

d. We cannot use the rescaling properties for $E(X - .01X^2)$, since this isn't a linear function of X. However, since we've already found both $E(X)$ and $E(X^2)$, we may as well use them: the expected actual capacity of a freezer is $E(X - .01X^2) = E(X) - .01E(X^2) = 16.38 - .01(272.298) = 13.657$ cubic feet. Alternatively, you can figure out the actual capacity for each of the three freezer types and take the weighted average.

33.

a. $E(X^2) = \sum_{x=0}^{1} x^2 \cdot p(x) = 0^2(1-p) + 1^2(p) = p$.

b. $V(X) = E(X^2) - [E(X)]^2 = p - [p]^2 = p(1-p)$.

c. $E(X^{79}) = 0^{79}(1-p) + 1^{79}(p) = p$. In fact, $E(X^n) = p$ for any non-negative power n.

49. Let X be the number of "seconds," so $X \sim \text{Bin}(6, .10)$.

a. $P(X=1) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{6}{1}(.1)^1(.9)^5 = .3543$.

b. $P(X \geq 2) = 1 - [P(X=0) + P(X=1)] = 1 - \left[\binom{6}{0}(.1)^0(.9)^6 + \binom{6}{1}(.1)^1(.9)^5\right] = 1 - [.5314 + .3543] =$ $.1143$.

c. Either 4 or 5 goblets must be selected.

Select 4 goblets with zero defects: $P(X=0) = \binom{4}{0}(.1)^0(.9)^4 = .6561$.

Select 4 goblets, one of which has a defect, and the 5th is good: $\left[\binom{4}{1}(.1)^1(.9)^3\right] \times .9 = .26244$

So, the desired probability is $.6561 + .26244 = .91854$.

50. Let X be the number of faxes, so $X \sim \text{Bin}(25, .25)$.

a. $P(X \leq 6) = B(6;25,.25) = .561$.

b. $P(X = 6) = b(6;25,.25) = .183$.

c. $P(X \geq 6) = 1 - P(X \leq 5) = 1 - B(5;25,.25) = .622$.

d. $P(X > 6) = 1 - P(X \leq 6) = 1 - .561 = .439$.

51. Let X be the number of faxes, so $X \sim \text{Bin}(25, .25)$.

a. $E(X) = np = 25(.25) = 6.25$.

b. $V(X) = np(1-p) = 25(.25)(.75) = 4.6875$, so $SD(X) = 2.165$.

c. $P(X > 6.25 + 2(2.165)) = P(X > 10.58) = 1 - P(X \leq 10.58) = 1 - P(X \leq 10) = 1 - B(10;25,.25) = .030$.

52. Let X be the number of students who want a new copy, so $X \sim \text{Bin}(n = 25, p = .3)$.

a. $E(X) = np = 25(.3) = 7.5$ and $SD(X) = \sqrt{np(1-p)} = \sqrt{25(.3)(.7)} = 2.29$.

b. Two standard deviations from the mean converts to $7.5 \pm 2(2.29) = 2.92$ & 12.08. For X to be more than two standard deviations from the means requires $X < 2.92$ or $X > 12.08$. Since X must be a non-negative integer, $P(X < 2.92 \text{ or } X > 12.08) = 1 - P(2.92 \le X \le 12.08) = 1 - P(3 \le X \le 12) =$

$$1 - \sum_{x=3}^{12} \binom{25}{x} (.3)^x (.7)^{25-x} = 1 - .9736 = .0264.$$

c. If $X > 15$, then more people want new copies than the bookstore carries. At the other end, though, there are $25 - X$ students wanting used copies; if $25 - X > 15$, then there aren't enough used copies to meet demand.

The inequality $25 - X > 15$ is the same as $X < 10$, so the bookstore can't meet demand if either $X > 15$ or $X < 10$. All 25 students get the type they want iff $10 \le X \le 15$:

$$P(10 \le X \le 15) = \sum_{x=10}^{15} \binom{25}{x} (.3)^x (.7)^{25-x} = .1890.$$

d. The bookstore sells X new books and $25 - X$ used books, so total revenue from these 25 sales is given by $h(X) = 100(X) + 70(25 - X) = 30X + 1750$. Using linearity/rescaling properties, expected revenue equals $E(h(X)) = E(30X + 1750) = 30\mu + 1750 = 30(7.5) + 1750 = \$1975$.