34.

a. Since order doesn't matter, the number of ways to randomly select 5 keyboards from the 25 available is $\binom{25}{5} = 53{,}130$.

b. Sample in two stages. First, there are 6 keyboards with an electrical defect, so the number of ways to select exactly 2 of them is $\binom{6}{2}$. Next, the remaining 5 – 2 = 3 keyboards in the sample must have mechanical defects; as there are 19 such keyboards, the number of ways to randomly select 3 is $\binom{19}{3}$. So, the number of ways to achieve both of these in the sample of 5 is the product of these two counting numbers: $\binom{6}{2}\binom{19}{3} = (15)(969) = 14{,}535$.

c. Following the analogy from **b**, the number of samples with exactly 4 mechanical defects is $\binom{19}{4}\binom{6}{1}$, and the number with exactly 5 mechanical defects is $\binom{19}{5}\binom{6}{0}$. So, the number of samples with <u>at least</u> 4 mechanical defects is $\binom{19}{4}\binom{6}{1} + \binom{19}{5}\binom{6}{0}$, and the probability of this event is

$$\frac{\binom{19}{4}\binom{6}{1}+\binom{19}{5}\binom{6}{0}}{\binom{25}{5}} = \frac{34{,}884}{53{,}130} = .657.$$ (The denominator comes from **a**.)

38.

a. There are 6 75W bulbs and 9 other bulbs. So, P(select exactly 2 75W bulbs) = P(select exactly 2 75W bulbs and 1 other bulb) $= \dfrac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \dfrac{(15)(9)}{455} = .2967$.

b. P(all three are the same rating) = P(all 3 are 40W or all 3 are 60W or all 3 are 75W) =
$\dfrac{\binom{4}{3}+\binom{5}{3}+\binom{6}{3}}{\binom{15}{3}} = \dfrac{4+10+20}{455} = .0747$.

c. P(one of each type is selected) $= \dfrac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}} = \dfrac{120}{455} = .2637$.

d. It is necessary to examine at least six bulbs if and only if the first five light bulbs were all of the 40W or 60W variety. Since there are 9 such bulbs, the chance of this event is
$\dfrac{\binom{9}{5}}{\binom{15}{5}} = \dfrac{126}{3003} = .042$.

45.

a. $P(A) =$ .106 + .141 + .200 = .447, $P(C)$ =.215 + .200 + .065 + .020 = .500, and $P(A \cap C)$ = .200.

b. $P(A|C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{.200}{.500} = .400$. If we know that the individual came from ethnic group 3, the probability that he has Type A blood is .40. $P(C|A) = \dfrac{P(A \cap C)}{P(A)} = \dfrac{.200}{.447} = .447$. If a person has Type A blood, the probability that he is from ethnic group 3 is .447.

c. Define D = "ethnic group 1 selected." We are asked for $P(D|B')$. From the table, $P(D \cap B')$ = .082 + .106 + .004 = .192 and $P(B') = 1 - P(B) = 1 - [.008 + .018 + .065]$ = .909. So, the desired probability is $P(D|B') = \dfrac{P(D \cap B')}{P(B')} = \dfrac{.192}{.909} = .211$.

46. Let A be that the individual is more than 6 feet tall. Let B be that the individual is a professional basketball player. Then $P(A|B)$ = the probability of the individual being more than 6 feet tall, knowing that the individual is a professional basketball player, while $P(B|A)$ = the probability of the individual being a professional basketball player, knowing that the individual is more than 6 feet tall. $P(A|B)$ will be larger. Most professional basketball players are tall, so the probability of an individual in that reduced sample space being more than 6 feet tall is very large. On the other hand, the number of individuals that are pro basketball players is small in relation to the number of males more than 6 feet tall.

47. A Venn diagram appears at the end of this exercise.

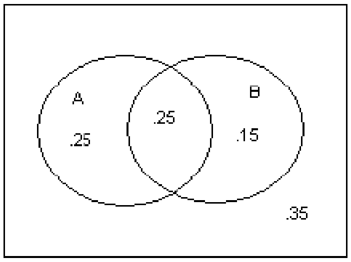
a. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.25}{.50} = .50$.

b. $P(B'|A) = \dfrac{P(A \cap B')}{P(A)} = \dfrac{.25}{.50} = .50$.

c. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.25}{.40} = .6125$.

d. $P(A'|B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{.15}{.40} = .3875$.

e. $P(A|A \cup B) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(A)}{P(A \cup B)} = \dfrac{.50}{.65} = .7692$. It should be clear from the Venn diagram that $A \cap (A \cup B) = A$.



64. A tree diagram can help. We know that P(short) = .6, P(medium) = .3, P(long) = .1; also, P(Word | short) = .8, P(Word | medium) = .5, P(Word | long) = .3.

a. Use the law of total probability: P(Word) = (.6)(.8) + (.3)(.5) + (.1)(.3) = .66.

b. $P(\text{small} \mid \text{Word}) = \dfrac{P(\text{small} \cap \text{Word})}{P(\text{Word})} = \dfrac{(.6)(.8)}{.66} = .727$. Similarly, $P(\text{medium} \mid \text{Word}) = \dfrac{(.3)(.5)}{.66} = .227$, and P(long | Word) = .045. (These sum to .999 due to rounding error.)

67. Let T denote the event that a randomly selected person is, in fact, a terrorist. Apply Bayes' theorem, using $P(T) = 1{,}000/300{,}000{,}000 = .0000033$:

$P(T \mid +) = \dfrac{P(T)P(+ \mid T)}{P(T)P(+ \mid T) + P(T')P(+ \mid T')} = \dfrac{(.0000033)(.99)}{(.0000033)(.99) + (1 - .0000033)(1 - .999)} = .003289$. That is to say, roughly 0.3% of all people "flagged" as terrorists would be actual terrorists in this scenario.

80. Let A_i denote the event that component #i works ($i = 1, 2, 3, 4$). Based on the design of the system, the event "the system works" is $(A_1 \cup A_2) \cup (A_3 \cap A_4)$. We'll eventually need $P(A_1 \cup A_2)$, so work that out first: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = (.9) + (.9) - (.9)(.9) = .99$. The third term uses independence of events. Also, $P(A_3 \cap A_4) = (.9)(.9) = .81$, again using independence.

Now use the addition rule and independence for the system:

$$\begin{aligned} P((A_1 \cup A_2) \cup (A_3 \cap A_4)) &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P((A_1 \cup A_2) \cap (A_3 \cap A_4)) \\ &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P(A_1 \cup A_2) \times P(A_3 \cap A_4) \\ &= (.99) + (.81) - (.99)(.81) = .9981 \end{aligned}$$

(You could also use deMorgan's law in a couple of places.)

83. We'll need to know P(both detect the defect) = 1 – P(at least one doesn't) = 1 – .2 = .8.

a. $P(1^{st}$ detects $\cap$ 2^{nd} doesn't) = $P(1^{st}$ detects) – $P(1^{st}$ does $\cap$ 2^{nd} does) = .9 – .8 = .1. Similarly, $P(1^{st}$ doesn't $\cap$ 2^{nd} does) = .1, so P(exactly one does)= .1 + .1= .2.

b. P(neither detects a defect) = 1 – [P(both do) + P(exactly 1 does)] = 1 – [.8+.2] = 0. That is, under this model there is a 0% probability neither inspector detects a defect. As a result, P(all 3 escape) = (0)(0)(0) = 0.

84. Let A_i denote the event that vehicle #i passes inspection ($i = 1, 2, 3$).

a. $P(A_1 \cap A_2 \cap A_3) = P(A_1) \times P(A_2) \times P(A_3) = (.7)(.7)(.7) = (.7)^3 = .343$.

b. This is the complement of part **a**, so the answer is 1 – .343 = .657.

c. $P([A_1 \cap A_2' \cap A_3'] \cup [A_1' \cap A_2 \cap A_3'] \cup [A_1' \cap A_2' \cap A_3]) = (.7)(.3)(.3) + (.3)(.7)(.3) + (.3)(.3)(.7) = 3(.3)^2(.7)$ = .189. Notice that we're using the fact that if events are independent then their complements are also independent.

d. P(at most one passes) = P(zero pass) + P(exactly one passes) = P(zero pass) + .189. For the first probability, P(zero pass) = $P(A_1' \cap A_2' \cap A_3') = (.3)(.3)(.3) = .027$. So, the answer is .027 + .189 = .216.

e. We'll need the fact that P(at least one passes) = 1 – P(zero pass) = 1 – .027 = .973. Then,

$$P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3) = \frac{P([A_1 \cap A_2 \cap A_3] \cap [A_1 \cup A_2 \cup A_3])}{P(A_1 \cup A_2 \cup A_3)} = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} = \frac{.343}{.973} = .3525.$$