45.

a. $\overline{x} = 115.58$. The deviations from the mean are $116.4 - 115.58 = .82$, $115.9 - 115.58 = .32$, $114.6 - 115.58 = -.98$, $115.2 - 115.58 = -.38$, and $115.8 - 115.58 = .22$. Notice that the deviations from the mean sum to zero, as they should.

b. $s^2 = [(.82)^2 + (.32)^2 + (-.98)^2 + (-.38)^2 + (.22)^2]/(5-1) = 1.928/4 = .482$, so $s = .694$.

c. $\Sigma x_i^2 = 66795.61$, so $s^2 = S_{xx}/(n-1) = \left(\Sigma x_i^2 - (\Sigma x_i)^2/n\right)/(n-1) =$ $(66795.61 - (577.9)^2/5)/4 = 1.928/4 = .482$.

d. The new sample values are: 16.4 15.9 14.6 15.2 15.8. While the new mean is 15.58, all the deviations are the same as in part (a), and the variance of the transformed data is identical to that of part (b).

46.

a. From software, $\overline{x} = 2887.6$ and $\tilde{x} = 2888$.

b. Subtracting a constant from each observation shifts the data, but does not change its sample variance. For example, by subtracting 2700 from each observation we get the values 81, 200, 313, 156, and 188, which are smaller (fewer digits) and easier to work with. The sum of squares of this transformed data is 204210 and their sum is 938, so the computational formula for the variance gives $s^2 = (204210 - (938)^2/5)/(5-1) = 7060.3$.

51.

a. From software, $s^2 = 1264.77 \text{ min}^2$ and $s = 35.56$ min. Working by hand, $\Sigma x = 2563$ and $\Sigma x^2 = 368501$, so

$$s^2 = \frac{368501 - (2563)^2/19}{19-1} = 1264.766 \text{ and } s = \sqrt{1264.766} = 35.564$$

b. If y = time in hours, then $y = cx$ where $c = \frac{1}{60}$. So, $s_y^2 = c^2 s_x^2 = \left(\frac{1}{60}\right)^2 1264.766 = .351 \text{ hr}^2$ and $s_y = cs_x = \left(\frac{1}{60}\right)35.564 = .593 \text{ hr}$.

52. Let d denote the fifth deviation. Then $.3 + .9 + 1.0 + 1.3 + d = 0$ or $3.5 + d = 0$, so $d = -3.5$. One sample for which these are the deviations is $x_1 = 3.8$, $x_2 = 4.4$, $x_3 = 4.5$, $x_4 = 4.8$, $x_5 = 0$. (These were obtained by adding 3.5 to each deviation; adding any other number will produce a different sample with the desired property.)

62. To simplify the math, subtract the mean from each observation; i.e., let $y_i = x_i - \overline{x} = x_i - 76831$. Then $y_1 = 76683 - 76831 = -148$ and $y_4 = 77048 - 76831 = 217$; by rescaling, $\overline{y} = \overline{x} - 76831 = 0$, so $y_2 + y_3 = -(y_1 + y_4) = -69$. Also,

$$180 = s = \sqrt{\frac{\Sigma(x_i - \overline{x})^2}{n-1}} = \sqrt{\frac{\Sigma y_i^2}{3}} \quad \Rightarrow \quad \Sigma y_i^2 = 3(180)^2 = 97200$$

so $y_2^2 + y_3^2 = 97200 - (y_1^2 + y_4^2) = 97200 - ((-148)^2 + (217)^2) = 28207$.

To solve the equations $y_2 + y_3 = -69$ and $y_2^2 + y_3^2 = 28207$, substitute $y_3 = -69 - y_2$ into the second equation and use the quadratic formula to solve. This gives $y_2 = 79.14$ or -148.14 (one is y_2 and one is y_3).

Finally, x_2 and x_3 are given by $y_2 + 76831$ and $y_3 + 76831$, or 79,610 and 76,683.

68.

a.

$$\frac{d}{dc}\left\{\sum (x_i - c)^2\right\} = \sum \frac{d}{dc}(x_i - c)^2 = -2\sum (x_i - c) = 0 \Rightarrow \sum (x_i - c) = 0 \Rightarrow$$

$$\sum x_i - \sum c = 0 \Rightarrow \sum x_i - nc = 0 \Rightarrow nc = \sum x_i \Rightarrow c = \frac{\sum x_i}{n} = \overline{x}$$

b. Since $c = \overline{x}$ minimizes $\Sigma(x_i - c)^2$, $\Sigma(x_i - \overline{x})^2 < \Sigma(x_i - \mu)^2$.

69.

a.

$$\overline{y} = \frac{\sum y_i}{n} = \frac{\sum (ax_i + b)}{n} = \frac{a\sum x_i + \sum b}{n} = \frac{a\sum x_i + nb}{n} = a\overline{x} + b$$

$$s_y^2 = \frac{\sum (y_i - \overline{y})^2}{n-1} = \frac{\sum (ax_i + b - (a\overline{x} + b))^2}{n-1} = \frac{\sum (ax_i - a\overline{x})^2}{n-1}$$

$$= \frac{a^2 \sum (x_i - \overline{x})^2}{n-1} = a^2 s_x^2$$

b.

$$x = {}^\circ\text{C},\ y = {}^\circ\text{F}$$

$$\overline{y} = \frac{9}{5}(87.3) + 32 = 189.14^\circ\text{F}$$

$$s_y = \sqrt{s_y^2} = \sqrt{\left(\frac{9}{5}\right)^2 (1.04)^2} = \sqrt{3.5044} = 1.872^\circ\text{F}$$

3.

a. $A = \{SSF, SFS, FSS\}$.

b. $B = \{SSS, SSF, SFS, FSS\}$.

c. For event C to occur, the system must have component 1 working (S in the first position), then at least one of the other two components must work (at least one S in the second and third positions): $C = \{SSS, SSF, SFS\}$.

d. $C' = \{SFF, FSS, FSF, FFS, FFF\}$.
$A \cup C = \{SSS, SSF, SFS, FSS\}$.
$A \cap C = \{SSF, SFS\}$.
$B \cup C = \{SSS, SSF, SFS, FSS\}$. Notice that B contains C, so $B \cup C = B$.
$B \cap C = \{SSS\ SSF, SFS\}$. Since B contains C, $B \cap C = C$.

4.

a. The $2^4 = 16$ possible outcomes have been numbered here for later reference.

	Home Mortgage Number			
Outcome	1	2	3	4
1	*F*	*F*	*F*	*F*
2	*F*	*F*	*F*	*V*
3	*F*	*F*	*V*	*F*
4	*F*	*F*	*V*	*V*
5	*F*	*V*	*F*	*F*
6	*F*	*V*	*F*	*V*
7	*F*	*V*	*V*	*F*
8	*F*	*V*	*V*	*V*
9	*V*	*F*	*F*	*F*
10	*V*	*F*	*F*	*V*
11	*V*	*F*	*V*	*F*
12	*V*	*F*	*V*	*V*
13	*V*	*V*	*F*	*F*
14	*V*	*V*	*F*	*V*
15	*V*	*V*	*V*	*F*
16	*V*	*V*	*V*	*V*

b. Outcome numbers 2, 3, 5, 9 above.

c. Outcome numbers 1, 16 above.

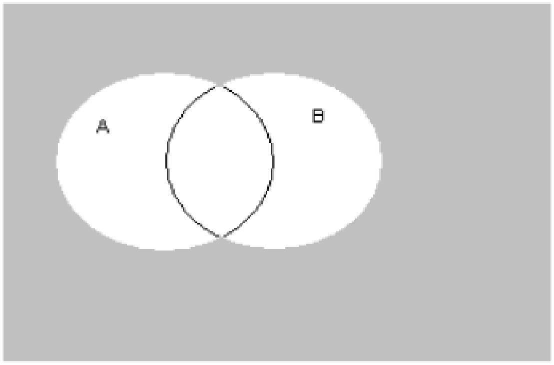
d. Outcome numbers 1, 2, 3, 5, 9 above.

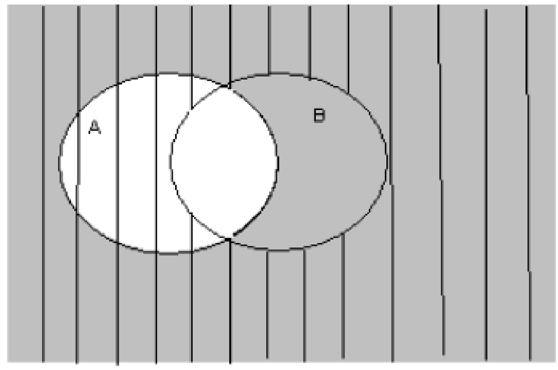
e. In words, the union of (c) and (d) is the event that either all of the mortgages are variable, or that at most one of them is variable-rate: outcomes 1, 2, 3, 5, 9, 16. The intersection of (c) and (d) is the event that all of the mortgages are fixed-rate: outcome 1.

f. The union of (b) and (c) is the event that either exactly three are fixed, or that all four are the same: outcomes 1, 2, 3, 5, 9, 16. The intersection of (b) and (c) is the event that exactly three are fixed and all four are the same type. This cannot happen (the events have no outcomes in common), so the intersection of (b) and (c) is $\varnothing$.

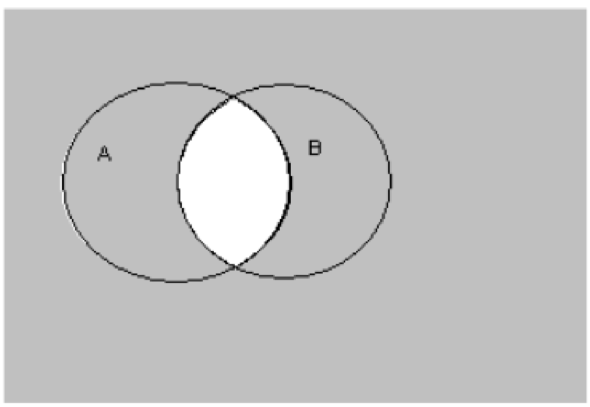
9.

a. In the diagram on the left, the shaded area is $(A \cup B)'$. On the right, the shaded area is A', the striped area is B', and the intersection $A' \cap B'$ occurs where there is both shading <u>and</u> stripes. These two diagrams display the same area.





b. In the diagram below, the shaded area represents $(A \cap B)'$. Using the right-hand diagram from (a), the <u>union</u> of A' and B' is represented by the areas that have either shading <u>or</u> stripes (or both). Both of the diagrams display the same area.



12.

a. $P(A \cup B) = .50 + .40 - .25 = .65$.

b. $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - .65 = .35$.

c. The event of interest is $A \cap B'$; from a Venn diagram, we see $P(A \cap B') = P(A) - P(A \cap B) = .50 - .25 = .25$.

13.

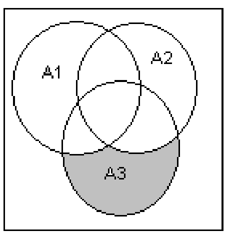
a. $A_1 \cup A_2$ = "awarded either #1 or #2 (or both)": from the addition rule,
$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = .22 + .25 - .11 = .36$.

b. $A_1' \cap A_2'$ = "awarded neither #1 or #2": using the hint and part (a),
$P(A_1' \cap A_2') = P((A_1 \cup A_2)') = 1 - P(A_1 \cup A_2) = 1 - .36 = .64$.

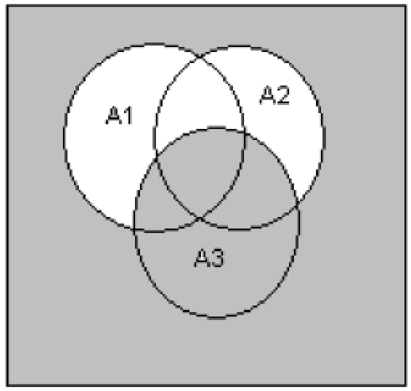
c. $A_1 \cup A_2 \cup A_3$ = "awarded at least one of these three projects": using the addition rule for 3 events,
$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) =$
$.22 + .25 + .28 - .11 - .05 - .07 + .01 = .53$.

d. $A_1' \cap A_2' \cap A_3'$ = "awarded none of the three projects":
$P(A_1' \cap A_2' \cap A_3') = 1 - P(\text{awarded at least one}) = 1 - .53 = .47$.

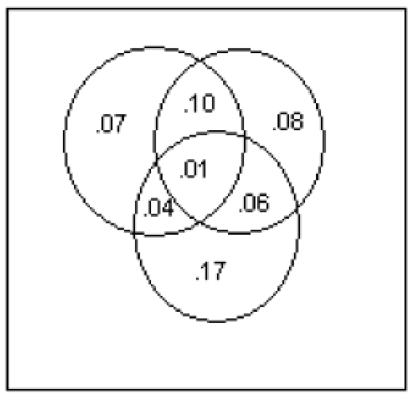
e. $A_1' \cap A_2' \cap A_3$ = "awarded #3 but neither #1 nor #2": from a Venn diagram,
$P(A_1' \cap A_2' \cap A_3) = P(A_3) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) =$
$.28 - .05 - .07 + .01 = .17$. The last term addresses the "double counting" of the two subtractions.



f. $(A_1' \cap A_2') \cup A_3$ = "awarded neither of #1 and #2, or awarded #3": from a Venn diagram, $P((A_1' \cap A_2') \cup A_3) = P(\text{none awarded}) + P(A_3) = .47$ (from **d**) $+ .28 = 75$.



Alternatively, answers to **a**-**f** can be obtained from probabilities on the accompanying Venn diagram:



14. Let A = an adult consumes coffee and B = an adult consumes carbonated soda. We're told that $P(A) = .55$, $P(B) = .45$, and $P(A \cup B) = .70$.

a. The addition rule says $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, so $.70 = .55 + .45 - P(A \cap B)$ or $P(A \cap B) = .55 + .45 - .70 = .30$.

b. There are two ways to read this question. We can read "does not (consume at least one)," which means the adult consumes neither beverage. The probability is then $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = 1 - P(A \cup B) = 1 - .70 = .30$.

The other reading, and this is presumably the intent, is "there is at least one beverage the adult does not consume, i.e. $A' \cup B'$. The probability is $P(A' \cup B') = 1 - P(A \cap B) = 1 - .30$ from **a** $= .70$. (It's just a coincidence this equals $P(A \cup B)$.)

Both of these approaches use *deMorgan's laws*, which say that $P(A' \cap B') = 1 - P(A \cup B)$ and $P(A' \cup B') = 1 - P(A \cap B)$.

17.

a. The probabilities do not add to 1 because there are other software packages besides SPSS and SAS for which requests could be made.

b. $P(A') = 1 - P(A) = 1 - .30 = .70$.

c. Since A and B are mutually exclusive events, $P(A \cup B) = P(A) + P(B) = .30 + .50 = .80$.

d. By deMorgan's law, $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - .80 = .20$.
In this example, deMorgan's law says the event "neither A nor B" is the complement of the event "either A or B." (That's true regardless of whether they're mutually exclusive.)

18. The only reason we'd need at least two selections to find a 75W bulb is if the first selection was not a 75W bulb. There are 6 + 5 = 11 non-75W bulbs out of 6 + 5 + 4 = 15 bulbs in the box, so the probability of this event is simply $\frac{11}{15}$.