Lecture Outlines:

- Large Sample Confidence Interval for a Population Proportion (Sec 7.2)
- Small Sample Confidence Interval for μ (Sec 7.3).

1. Large Sample Confidence Interval for a Population Proportion (Sec 7.2).

Let p denote the proportion of "success" in a population (i.e., the proportion of individuals who graduated from college). A random sample of n individuals is to be selected, and X is the number of successes in the sample. The point estimate of p is the sample proportion $\hat{p}$. When n is large, $\hat{p}$ approximately has the normal distribution with mean p and variance $p(1-p)/n$. From this fact, the $(1-\alpha)100\%$ confidence interval can be derived as (please read p.266 of the textbook for details)

$\hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})}$.

2. Small Sample Confidence Interval for μ

Here we assume that a random sample $X_1, \dots X_n$is from a normal distribution $N(\mu, \sigma^2)$ and the sample size $n < 30$. We will use the following fact to construct the confidence interval for μ:

$$T = \frac{(\bar{X} - \mu)}{\left(\frac{S}{\sqrt{n}}\right)}$$

has a t $-$distribution with $(n-1)$ degrees of freedom. The critical values t_α are provided in Table A.5. It follows that the $(1-\alpha)100\%$ confidence interval is

$$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

Note that the critical value $t_{\frac{\alpha}{2}}$ corresponds to $(n-1)$ degrees of freedom. For example, for a sample size 16, the critical value for the 95% confidence interval is $t_{0.025} = 2.131$ from Table A.5.

Please read Example 7.11 on page 273.