Computers & Geosciences 36 (2010) 512–519

Contents lists available at ScienceDirect

Computers & Geosciences

journal homepage: www.elsevier.com/locate/cageo

Interpolation algorithm ranking using cross-validation and the role of smoothing effect. A coal zone example ☆

Oriol Falivene [a,*,1], Lluís Cabrera [a], Raimon Tolosana-Delgado [b], Alberto Sáez [a]

[a] Geomodels Institute, Group of Geodynamics and Basin Analysis, Dept. EPGM, Universitat de Barcelona, c/Martí i Franquès s/n, Facultat de Geologia, 08028 Barcelona, Spain
[b] Maritime Engineering Laboratory, Technical University of Catalunya, c/Jordi Girona 1-3, 08034 Barcelona, Spain

ARTICLE INFO

Article history:
Received 17 February 2009
Received in revised form
28 September 2009
Accepted 30 September 2009

Keywords:
Interpolation
Cross-validation
Smoothing effect
Kriging
Inverse distance weighting

ABSTRACT

For a property measured at several locations, interpolation algorithms provide a unique and smooth function yielding a locally realistic estimation at any point within the sampled region. Previous studies searching for optimal interpolation strategies by measuring cross-validation error have not found consistent rankings; this fact was traditionally explained by differences in the distribution, spatial variability and sampling patterns of the datasets. This article demonstrates that ranking differences are also related to interpolation smoothing, an important factor controlling cross-validation errors that was not considered previously. Indeed, smoothing in average-based interpolation algorithms depends on the number of neighbouring data points used to obtain each interpolated value, among other algorithm parameters. A 3D dataset of calorific value measurements from a coal zone is used to demonstrate that different algorithm rankings can be obtained solely by varying the number of neighbouring points considered (i.e. whilst maintaining the distribution, spatial variability and sampling pattern of the dataset). These results suggest that cross-validation error cannot be used as a unique criterion to compare the performance of interpolation algorithms, as has been done in the past, and indicate that smoothing should be also coupled to search for optimum and geologically realistic interpolation algorithms.

1. Introduction

Interpolation algorithms aim to predict the value of a property at a location by using values of the same property sampled at scattered neighbouring points (Journel and Huijbregts, 1978; Jones et al., 1986; Davis, 2002). These algorithms yield a unique (though different for each method) property map honouring input data. Interpolation in geosciences is widely used for both predictive and visualization purposes. A variety of algorithms have been developed to carry out interpolations (Morrison, 1974), for example inverse distance weighting (IDW, Kane et al., 1982), Kriging, (Matheron, 1963), splines (Ahlberg et al., 1967; Mitasova and Mitas, 1993) or polynomial regression.

The selection of optimal interpolation strategies for continuous variables is an important and ongoing subject of debate (Lu and Wong, 2008; Bater and Coops, 2009). Cross-validation (CV) has often been used to compare the performance of interpolation algorithms (Table 1). CV is based on calculating the value of the variable at locations where the true value is known, but has been temporally removed from the input data, and then measuring the CV error by comparing the estimated value against the true one (Davis, 1987; Isaaks and Srivastava, 1989). Past comparisons based on CV error have yielded a variety of results, not always consistent (Table 1). For instance, in comparison of two widely used algorithms such as Kriging and IDW, some authors have found that Kriging yields better interpolations (Weber and Englund, 1994; Zimmerman et al., 1999; Goovaerts, 2000; Teegavarapu and Chandramouli, 2005; Lu and Wong, 2008), some have not found any significant differences in the results (Dirks et al., 1998; Moyeed and Papritz, 2002; Gallichand and Marcotte, 1993), and others have found that IDW yields better interpolations (Weber and Englund, 1992; Lu and Wong, 2008).

The disparity in the results obtained from existing interpolation algorithm rankings using CV error (Table 1) motivated this research. We demonstrate that the comparisons solely based on CV error are utterly flawed. Apart from the fact that rankings may depend on some specific characteristics of the particular dataset used for the comparison, we provide evidence that the size of the search neighbourhood plays a determinant role in algorithm rankings considering only CV error. The search neighbourhood is amongst the factors controlling the smoothing effect of each interpolation strategy. These findings challenge the practice of ranking and qualifying interpolation algorithms considering CV error (Table 1), and show that there is no absolute best

☆ Supplementary material is available to access through the online version.
* Corresponding author. Tel./Fax: +31(0) 70447 2336.
E-mail address: oriol.falivene@shell.com (O. Falivene).
[1] Present address: Shell International Exploration and Production, Kessler Park 1, 2280 AB Rijswijk, The Netherlands.

0098-3004/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.cageo.2009.09.015

Table 1
Summary of results from published interpolation algorithm comparisons by means of cross-validation (CV) check.

Interpolated property	Interpolation methods[a]	Comparison criteria	Results	Source
Surface elevation	Universal Kriging (24) and splines	Mean squared CV error[b]	Kriging provides lowest errors. Splines is less accurate but faster	Dubrule (1984)
Coal thickness	Ordinary Kriging (10–20) and Kriging with trend (10–20)	CV error	In interpolation conditions, both methods achieved the same results	Journel and Rossi (1989)
Digital elevation model transformed to simulate pollutant concentrations	Ordinary Kriging (4) and nearest neighbour	Mean absolute and squared CV error[c]	Kriging provides the lowest error and bias	Isaaks and Srivastava (1989, p. 353)
Horizontal permeability	Arithmetic mean, splines inverse squared distance weighting, and Kriging	Mean percentage CV error	–	Brummert et al. (1991)
Vertical–horizontal permeability ratio			Averaging provides the lowest errors	
Reservoir thickness			Splines provides the lowest errors	
Reservoir porosity			Kriging provides the lowest errors	
Local terrain elevation variance	Ordinary Kriging (20), inverse distance weighting, and inverse distance squared weighting [d].	Mean squared CV error	Inverse distance weighting provides the lowest errors	Weber and Englund (1992)
Subsurface clay content	Closest neighbour, inverse distance weighting (4 to 24), inverse squared distance weighting (4 to 24), Kriging (4 to 24)[e]	Mean absolute CV error	All gave similar results. Closest neighbour method yields the largest differences.	Gallichand and Marcotte (1993)
Terrain elevation and local terrain elevation variance	Ordinary Kriging (4, 12 and 20), inverse distance weighting (4, 12 and 20)[f] and splines.	Mean squared CV error	Kriging provides the lowest errors. Inverse distance weighting is sensitive to the number of neighbours averaged.	Weber and Englund (1994)
Rainfall records	Nearest neighbour, mean mapping, inverse-distance weighting[g] and Kriging	Root-mean squared CV error	All gave similar results. Inverse distance weighting results are the more realistic.	Dirks et al. (1998)
Synthetic variables	Ordinary Kriging, universal Kriging and inverse squared distance weighting (6 and 12)	Mean squared CV error[h]	Universal and ordinary Kriging provides the lowest errors	Zimmerman et al. (1999)
Averaged monthly rainfall records	Thiessen polygons, inverse distance weighting (16) and ordinary Kriging (16)[d]	Mean squared CV error	Kriging provides the lowest errors	Goovaerts (2000)
Co and Cu topsoil concentrations	Ordinary Kriging, lognormal Kriging, disjunctive Kriging, median indicator Kriging and model-based Kriging	Relative mean square CV error[d]	Precision of all the methods was practically the same	Moyeed and Papritz (2002)
Surface elevation	Nearest neighbour, splines and triangulation	CV error[i]	Splines and nearest neighbour provide the lowest errors	Okubo et al. (2004)
Rainfall records	Inverse distance weighting, ordinary Kriging[d]	Root-mean squared CV error[d]	Kriging provide the lowest errors	Teegavarapu and Chandramouli (2005)
Rainfall records and surface elevation	Inverse distance weighting, Kriging[d]	CV error[j]	Inverse distance weighting yield the lowest errors when poor spatial correlation (depends on power factor). Kriging provide the lowest errors when good spatial correlation	Lu and Wong (2008)
Surface elevation	Inverse distance weighting, natural neighbour, splines[d]	Mean absolute CV error[d,j]	Similar results, natural neighbour favoured for simplicity	Bater and Coops (2009)

[a] When available/known, the number of neighbouring points averaged to obtain each estimate is provided in brackets.
[b] Does not consider the cross-validated points for variogram calculation (i.e. jackknife, Deutsch and Journel, 1998).
[c] Uses also a linear loss function based on remediation economics.
[d] Among others.
[e] Among others.
[f] Power factor varying from 1 to 3.
[g] Power factor varying from 1 to 10.
[h] Variation of cross-validation. Computes the mean squared interpolation error obtained from comparing the interpolated results constrained by only a few samples from the complete synthetic variable, and the 'true' synthetic variable.
[i] Does not consider the cross-validated points for variogram calculation (i.e. jackknife, Deutsch and Journel, 1998).
[j] Distinguishes between estimation and validation subsets.

interpolation algorithm: one has to establish a trade-off between minimum CV error and predictions with low smoothing. A representative example, derived from a real 3D dataset with calorific values measured at wells from a coal mine, is used for illustration purposes (Fig. 1).

2. Methods

For our rankings, we considered two commonly used interpolation algorithms: IDW and Ordinary Kriging. Both methods provide an estimate Z^* of the studied variable $Z(x_0)$ at an unsampled location x_0, by means of a linear combination of N observed values of Z, denoted as $z_1, z_2, \ldots, z_N$,

$$Z^*(x_0) = \sum w_i z_i \tag{1}$$

For both algorithms compared, several numbers of averaged neighbours, N, ranging from 1 (nearest neighbour) to 288 were considered. Apart from well data locations (Fig. 1B), interpolations were also carried out over the whole three-dimensional grid (Fig. 1D) to attach a visual representation to the interpolation strategies compared by CV.

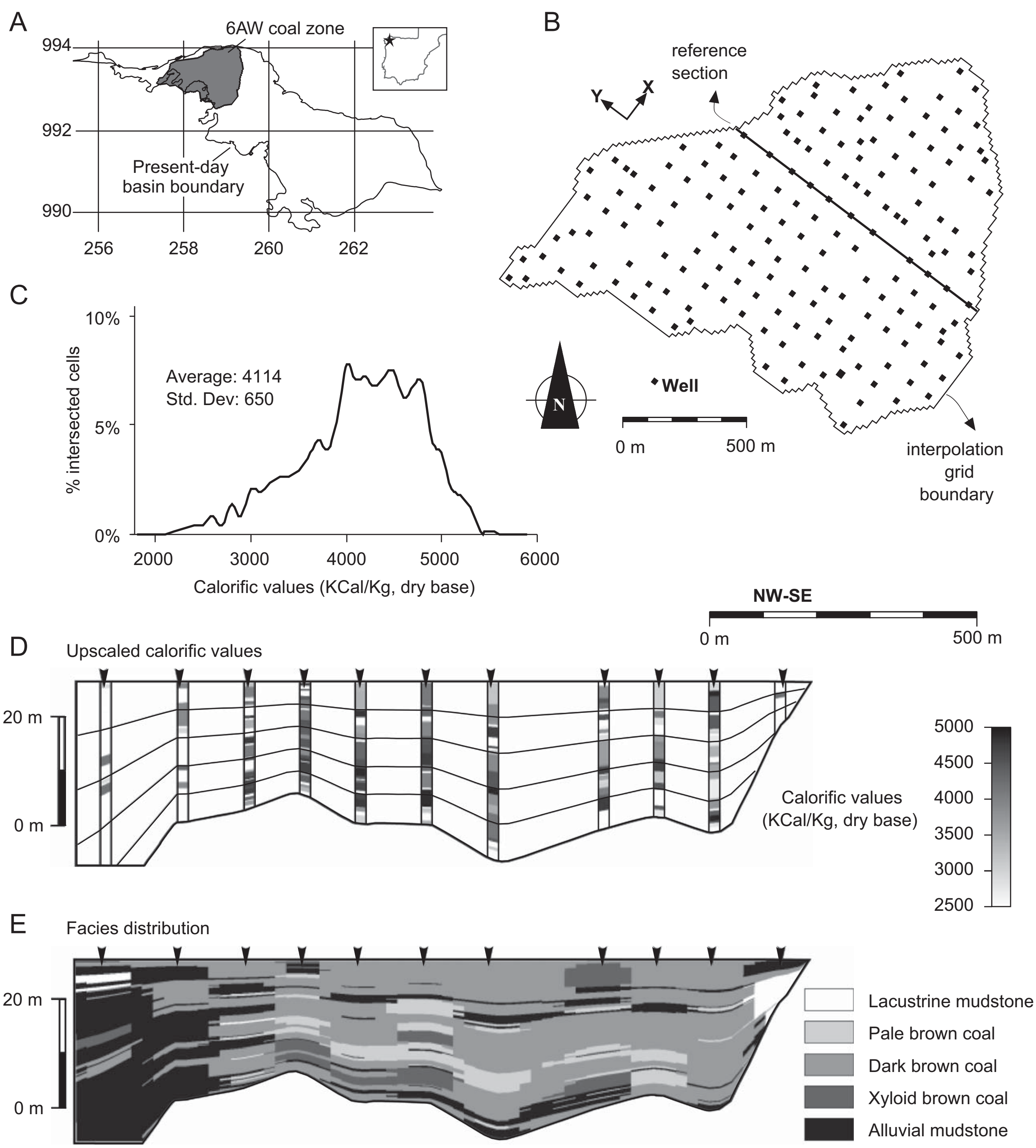

Fig. 1. Geological setting and dataset characteristics. (A) Present basin boundary and areal extent of 6AW interval. Coordinates are in kilometres; see basin location in upper right inset. (B) Well distribution in 6AW interval. Location of reference section in frames D and E and in Fig. 4 is shown. (C) Relative frequency of calorific values; plotted information corresponds to core data upscaled to the size of grid cells. (D) Reference section showing upscaled calorific values in intersected wells; calorific values in lacustrine and alluvial mudstones are null. Approximate paleodepositional surfaces are shown. (E) Facies distribution in 6AW obtained by using indicator Kriging with an areal trend applied to categorical variables (for details, see Falivene et al., 2007a). Vertical exaggeration of frames D and E is 10 ×.

IDW is a straightforward and simple interpolation method, in which the weights w_i of Eq. (1) for each averaged neighbouring data point are assigned according to an inverse of distance criterion (Kane et al., 1982).

$$w_i = \beta^{-1} d^{\alpha}(x_i, x_0), \quad \text{where } \beta = \sum d^{\alpha}(x_i, x_0)$$

Several distance weighting power factors were tested (α=1, 2 and 5). For the IDW interpolations the implementation in GSTAT was used (Pebesma and Wesseling, 1998).

Kriging is a geostatistical interpolation method in which the weights for each averaged neighbouring data point are defined to minimise the estimation variance (Matheron, 1963; Journel and Huijbregts, 1978; Cressie, 1990). The minimisation of this variance enables a spatial covariance criterion to be introduced, which results in weights for each data point that not only depend on the distance and direction to the grid cell being estimated (as in IDW), but also on the characteristics of the interpolated property (described by the variogram, $V(h)$, Fig. 2) and the relative positions of the averaged hard data (redundancy factor). For the Kriging interpolations the implementation in GSLIB was used (Deutsch and Journel, 1998).

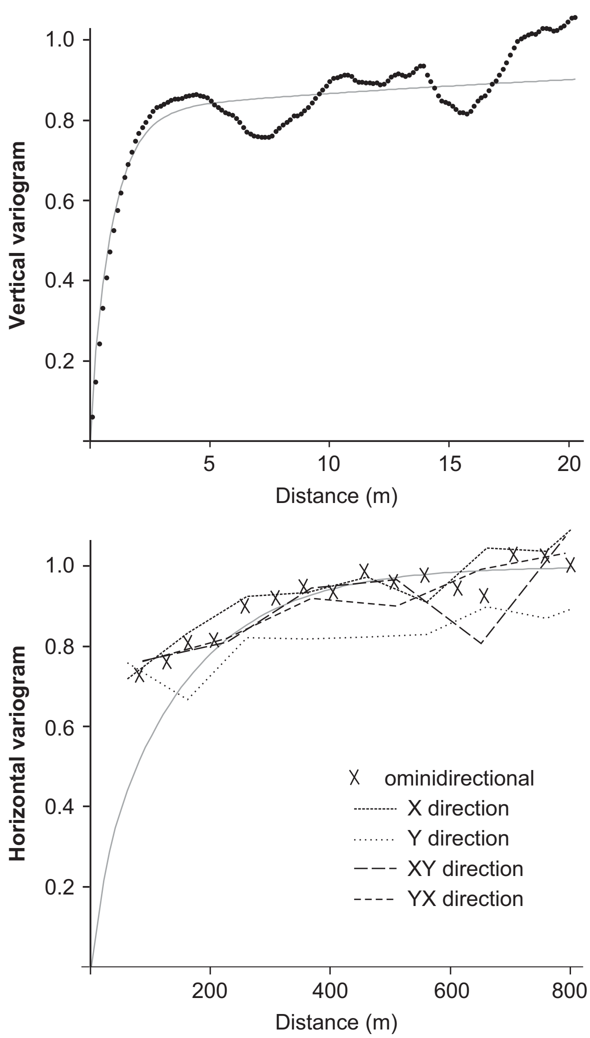

Fig. 2. Variograms for transformed calorific values. Black dots, crosses and dashed curves correspond to experimental variograms derived from upscaled well data. Grey continuous curves to theoretical models fitted with two exponential structures (*Hr* and *Vr* stand for horizontal and vertical ranges, respectively): $V(h)$=0.82 · Exp (*Hr*=450 m, *Vr*=2.8 m)+0.18 · Exp (*Hr*=60 m, *Vr*=100 m).

As usual when dealing with well data, CV was carried out by temporarily removing an entire well from the dataset (Deutsch, 2002), but using the model parameters derived from the exhaustive dataset to execute interpolations. CV error was taken as the average of the absolute differences between each predicted interpolation estimate and its corresponding real value. Standard deviation of the CV estimates was used to measure interpolation smoothing; their relationship is inverse (the higher the standard deviation, the lower the smoothing). Reference behaviours for the CV comparisons were defined by nearest neighbour interpolation, and random-based interpolation (i.e. assigning random values from the input distribution (Fig. 1C) considering different degrees of smoothing and without considering the neighbouring data points preferentially).

3. Illustration

3.1. Dataset, interpolation grid and interpolation parameters

The dataset used for illustration derives from the As Pontes Basin (NW Spain), a small mined non-marine basin (12 km^2) resulting from the activity of an Oligocene-Early Miocene strike-slip fault system (Bacelar et al., 1988; Santanach et al., 2005; Fig. 1A). The sedimentary basin fill consists of a 350–400 m thick succession of siliciclastic facies assemblages alternating and interfingering with coal deposits (Cabrera et al., 1995, 1996; Falivene et al., 2007a, 2007b), and was extensively drilled owing to coal mining interest. Lithofacies of the continuously cored exploration wells were correlated, taking into account the settling and spreading of the major coal seams, which are bounded by isochronous or near-isochronous surfaces. Several composite sequences and intervals were identified (Ferrús, 1998; Sáez and Cabrera, 2002; Sáez et al., 2003). Dry-base calorific values sampled on coal beds in 174 wells drilled through a 30 m thick, on average, coal-dominated interval (named 6AW, Falivene et al., 2007a) were used as the input data for the example in this study (Figs. 1B and C). These wells were drilled along a roughly square grid at a spacing of about 105 m. Original data consisted of more than 2700 calorific value analyses spread over 4000 m of recovered core. Calorific value distribution in these coals, which form laterally continuous beds of up to several hundreds of meters, is mainly influenced by the amount of detritic material, and shows gradual lateral variations (Figs. 1D and E).

To restore the post-depositional structural deformation (Santanach et al., 2005) and allow an easier visualization of calorific value distribution, interpolations were carried out with shifted vertical coordinates transforming the top of the 6AW zone to a horizontal datum. A grid layering combining proportional and parallel-to-the-top layering schemes was designed to mimic paleodepositional surfaces, along which facies and calorific values display the largest continuity (Fig. 1D). Horizontal grid spacing was set to 20 m. Vertical cell thickness was approximately 0.15 m, in line with the resolution of core descriptions. Calorific values measured in the cores were upscaled to the size of grid cells by arithmetic averaging (Fig. 1C), which averaged variability at smaller scales than the cell size. Upscaled calorific values measured in the coal beds were then transformed to a normal distribution using a normal-scores transformation (Deutsch and Journel, 1998). The transformed data were the input for further analyses.

Parameters required for interpolation algorithms (i.e. variogram parameters for Ordinary Kriging and vertical-to-horizontal anisotropy ratios for IDW) were adjusted from the complete dataset (Fig. 2). Anisotropy ratio (Jones et al., 1986; Falivene et al., 2007a) for IDW was approximated by the vertical-to-horizontal variogram range ratio. This factor is used to multiply the vertical coordinates prior to the interpolation in order to deal with geometric anisotropy (Kupfersberger and Deutsch, 1999). This enables assigning different weights to hard data points located at the same real distance from the point being estimated, but with different stratigraphic position, and allows reproducing flattened geometries, which are typical of sedimentary deposits.

3.2. Results

Results were computed directly both for the normal property and after undoing the normal-scores transformation to the original data scale. As both results are qualitatively similar, for simplicity and geological relevance only the back-transformed results are shown (Figs. 3–5). Results in Fig. 3 can be sum marized as

(1) CV error is not independent on smoothing; for random-based interpolation, as smoothing increases, CV error decreases (Fig. 3). Nearest neighbour interpolation yields the largest CV error and the lowest smoothing with respect to Kriging and IDW (Fig. 3).
(2) Compared to the results of random-based interpolation, by using average-based interpolation methods, CV error and smoothing are always smaller than by nearest neighbour interpolation (Fig. 3).
(3) When a small number of neighbouring data points are considered (Fig. 4A and B), the largest CV errors are obtained (Fig. 3). If the number of neighbouring data points increases (Fig. 4C and D), then CV error decreases (Fig. 3). In IDW, for very large numbers of neighbouring points, CV error increases slightly.
(4) Smoothing always increases as the number of neighbours increases (Herzfeld et al., 1993, Fig. 3).
(5) For IDW, on increasing the power factor, smoothing decreases, whereas CV error tends to increase (Fig. 3B and C). Increasing the power factor increases the importance of the nearest samples, thus effectively reducing the number of influential samples in the neighbourhood.
(6) Depending on the degree of interpolation smoothing (i.e. on the number of neighbours considered for interpolation), completely different algorithm rankings can be obtained if only CV error is taken into account (Fig. 3B and C).

4. Discussion and conclusions

An optimal interpolation algorithm should provide minimum cross-validation (CV) error, as is common practice in the literature (Table 1). CV errors in the example presented here range between 10% and 15% of the mean measured calorific value (Fig. 3). These variations are large enough to rank the different algorithms, and can be significant when predictions are made over large coal volumes. In addition, an optimal interpolation algorithm should also obtain results with relatively low interpolation smoothing (Isaaks and Srivastava, 1989; Olea and Pawlowsky, 1996; Journel et al., 2000), which seeks to preserve as much as possible the gradual lateral variation of calorific values shown in the mine (Fig. 1D, compare Figs. 4A–C, and B–D, Fig. 5).

Variations in interpolation algorithm rankings, taking only measurements of CV error (Table 1) have been traditionally

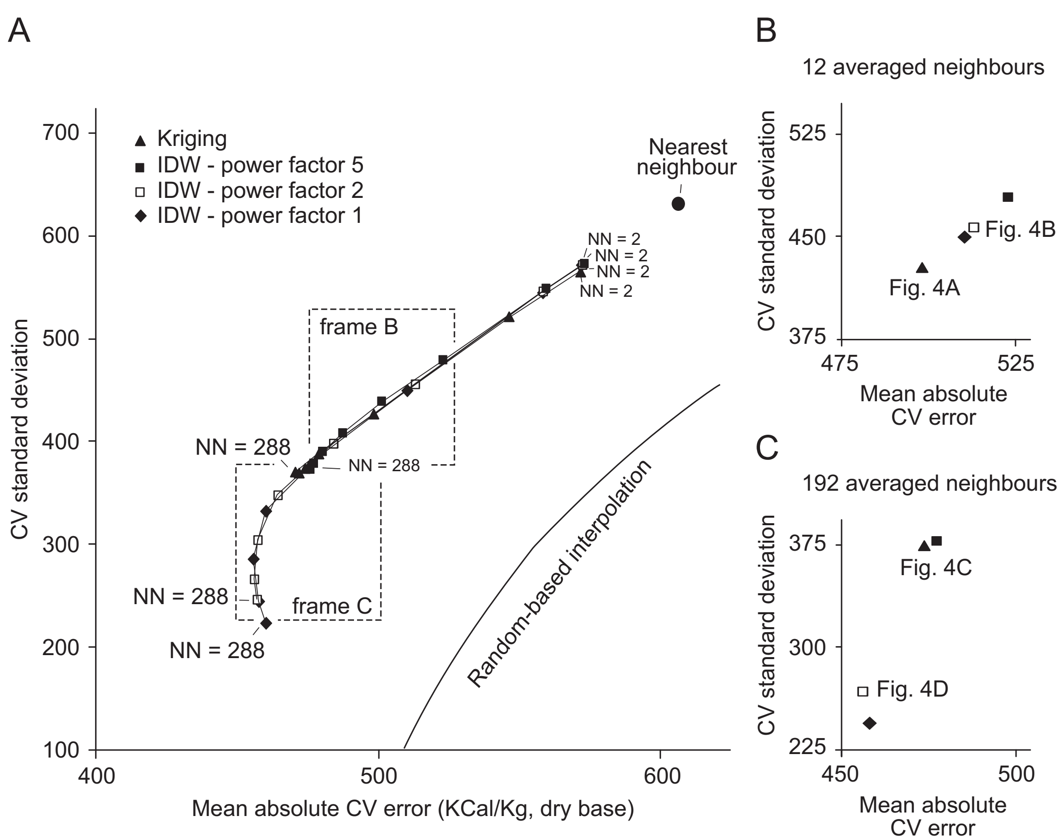

Fig. 3. Interpolation smoothing (measured by standard deviation of cross validation (CV) estimates) against mean absolute CV error for all interpolation strategies compared. The greater the standard deviation, the lower the smoothing; standard deviation in the original dataset was 650. (A) Results for several numbers of averaged neighbours (2, 4, 12, 24, 48, 96, 192 and 288). Note also results of nearest neighbour and random-based interpolations (i.e. assigning random values from the input distribution (with different smoothing degrees), and without considering neighbouring points). (B) Detail with results for 12 averaged neighbours. (C) Detail with results for 192 averaged neighbours. Note correspondences with frames in Fig. 4.

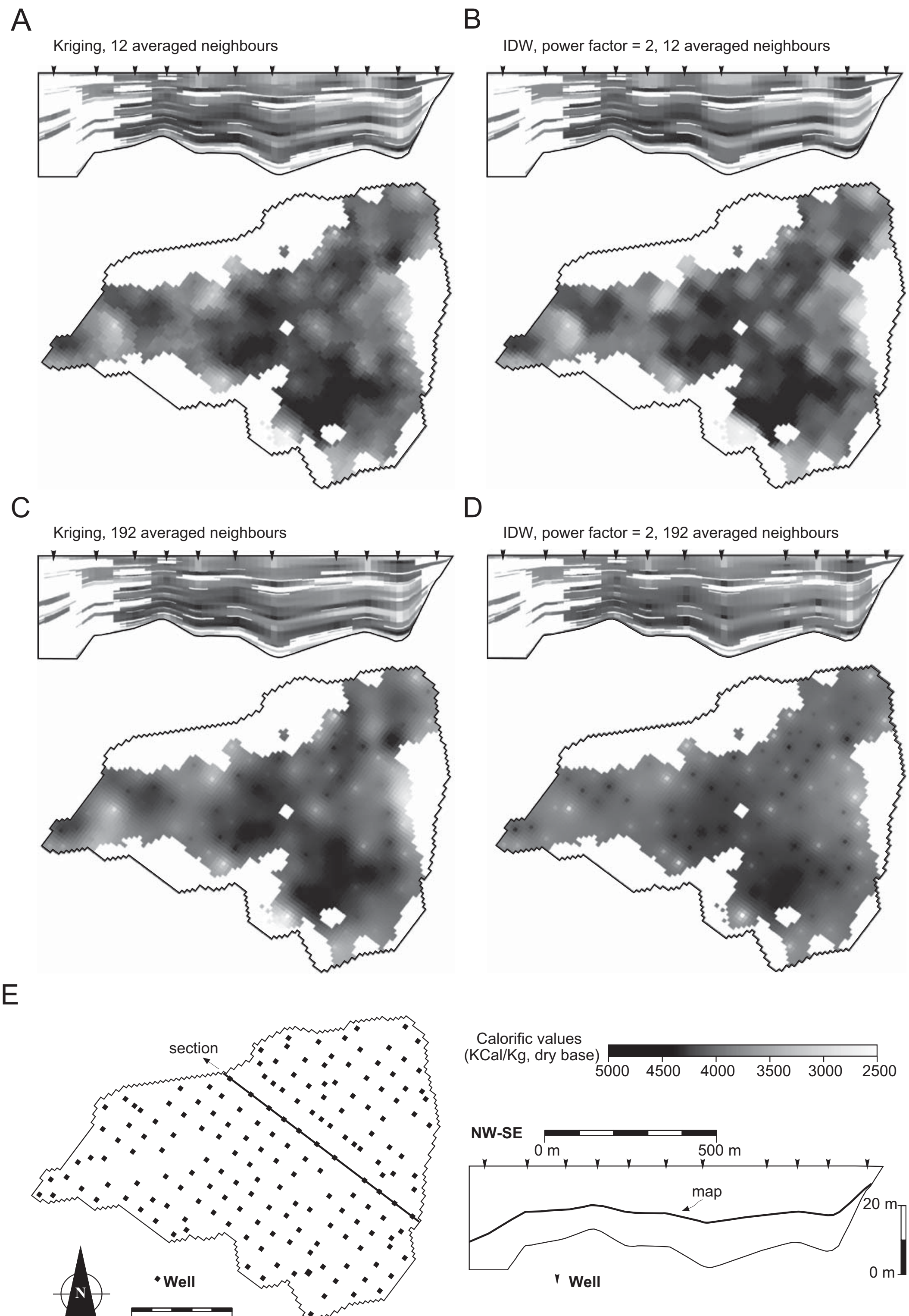

Fig. 4. (A, B, C and D) Reference section and map showing calorific value distributions in coal facies obtained by different interpolation strategies. Calorific value in alluvial and lacustrine mudstone facies shown in Fig. 1E is null. (E) Location of reference section, map and input well data. Note that the horizontal scale of the map and the section are not the same. If the number of averaged neighbours increases, spatial continuity of resultant calorific value distribution in coal facies is obscured, as the result of larger interpolation smoothing. Vertical exasggeration 10 ×.

justified by the fact that the studied variables are characterized by different histogram distributions, spatial continuity or sampling patterns (Brummert et al., 1991; Zimmerman et al., 1999; Lu and Wong, 2008). For example, a general consensus exists that, in irregularly spaced data, Kriging should provide more accurate and robust results than IDW, because Kriging takes into account the relative positions of sampling points, and not only their dis tance from the interpolated point (Kane et al., 1982; Lebel et al., 1987; Weber and Englund, 1994; Borga and Vizzaccaro, 1997; Goovaerts, 2000; Falivene et al., 2007a).

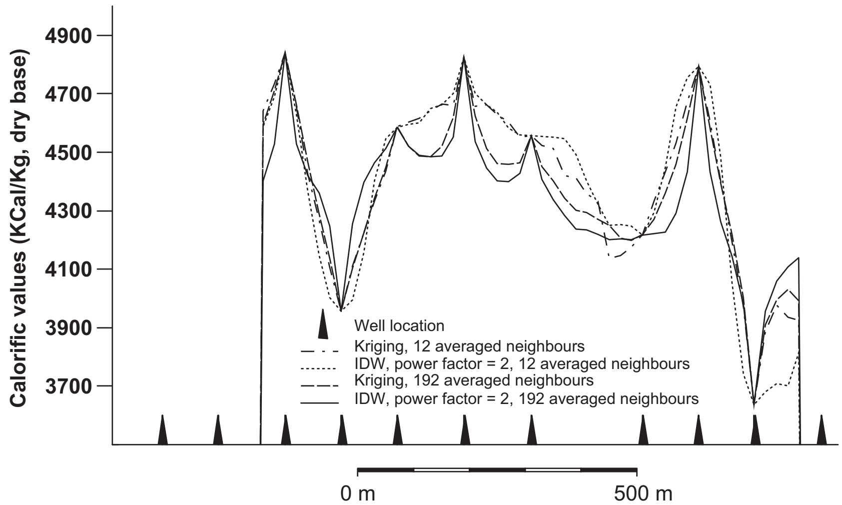

Fig. 5. Calorific values for cells in the intersection of map and section in Fig. 4, obtained by different interpolation strategies. Note that too smooth interpolation methods such as Kriging or IDW with 192 averaged neighbours provide interpolations that in some cases deviate largely from closest surrounding data due to influence of data located further away, although they yield lower CV errors than algorithms considering a smaller number of averaged neighbours.

The results shown herein demonstrate that, if only CV error is considered, different algorithm rankings can be obtained by changing the number of neighbours averaged (Figs. 3B and C). Thus, differences in algorithm rankings cannot be fully explained by intrinsic differences related to the variable studied and the sampling patterns, as suggested before. Indeed, interpolation smoothing partially controls the results of CV error (Fig. 3). Interpolation smoothing is primarily controlled by the number of neighbours averaged, but also by the algorithm itself and other algorithm parameters (e.g. the semivariogram in Kriging and the anisotropy ratio and the power factor in inverse distance weighting).

As a consequence, using only CV error as ranking criteria provides ambiguous results, because smoothing (relating to each particular algorithm and algorithm parameters) heavily influences the CV rankings and the appearance and continuity of the interpolation results (Figs. 4 and 5). The interpolation results obtained with the largest number of neighbours are the ones that yield the lowest CV error, but Figs. 4 and 5 show that the predictions between data points in these cases tend to be too smooth, because of the increasing influence from too much data further away. Therefore, minimum CV error cannot be the unique criterion of interpolation optimality, as used in previous studies (Table 1). Even for the same inter polation method, the optimum number of neighbours averaged is not the one that yields minimum CV errors because the smoothing introduced in the interpolation must also be taken into account.

Multiple-criterion rankings, for instance coupling CV error and smoothing, need to be used to look for optimum interpolation strategies. This multi-criterion would discard too smooth calorific value distributions (i.e. disconnecting large and small calorific values identified in adjacent wells, such as those in Fig. 4D), even though they may yield the lowest CV error (Fig. 3C); and it would highlight distributions with gradual and laterally continuous calorific values, with moderate CV error and smoothing (such as those in Figs. 4A or B, Fig. 5), even though they may not yield the lowest CV error (Fig. 3B). Therefore, in more general terms applicable to other geological situations or case studies, the analyst should search for a trade-off between geological continuity (low smoothing) and statistical optimality (low average CV error), in order to look for the best interpolation practices.

Acknowledgements

Financial support from the Generalitat de Catalunya (Grup de Recerca de Geodinàmica i Anàlisi de Conques, 2005SGR-000397) and from the MCyT is acknowledged (projects CGL2007-66431-C02-01/BTE (MARES 3D/4D) and CGL2007-66431-C02-02/BTE (REMOSS 3D/4D)). ENDESA MINA PUENTES is thanked for providing the dataset. Roxar is thanked for providing the IRAP RMS reservoir modelling software. RT acknowledges funding through the research projects BPM2003-05640 (MESS) and MTM2006-03040 (MEASURE) of the MEC for the research group on statistics and data analysis (University of Girona). Journal reviewers (Dr. O. Dubrule and two anonymous reviewers) and editor (C.V. Deutsch) are thanked for their constructive remarks.

Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.cageo.2009.9.015.

References

Ahlberg, J.H., Nilson, E.W., Walsh, J.L., 1967. The Theory of Splines and its Applications. Academic Press, New York 280 pp.

Bacelar, J., Alonso, M., Kaiser, C., Sanchez, M., Cabrera, L., Sáez, A., Santanach, P., 1988. La Cuenca Terciaria de As Pontes (Galicia): su desarrollo asociado a inflexiones contractivas de una falla direccional (The As Pontes Teriary Basin (Galicia): development linked to restraining bends in a strike-slip fault). In: Proceedings II Congreso Geológico de España, Sociedad Geologica de España, Granada, pp. 113–121.

Bater, C.W., Coops, N.C., 2009. Evaluating error associated with lidar-derived DEM interpolation. Computers & Geosciences 35, 289–300.

Borga, M., Vizzaccaro, A., 1997. On the interpolation of hydrologic variables: formal equivalence of multiquadratic surface fitting and kriging. Journal of Hydrology 195, 160–171.

Brummert, A.C., Pool, S.E., Portman, M.E., Hancock, J.S., Ammer, J.R., 1991. Determining optimum estimation methods for interpolation and extrapolation of reservoir properties: a case study. In: Lake, L.W., Carroll, H.B., Wesson, T.C. (Eds.), Reservoir Characterization. Academic Press, New York, pp. 445–485.

Cabrera, L., Ferrús, B., Sáez, A., Santanach, P., Bacelar., J., 1996. Onshore Cenozoic strike-slip basins in NW Spain. In: Friend, P.F., Dabrio, C.J. (Eds.), Tertiary Basins of Spain, the Stratigraphic Record of Crustal Kinematics. University Press, Cambridge, pp. 247–254.

Cabrera, L., Hagemann, H.W., Pickel, W., Sáez, A., 1995. The coal-bearing, Cenozoic As Pontes basin (northwestern Spain): geological influence on coal characteristics. International Journal of Coal Geology 27, 201–226.

Cressie, N., 1990. The origins of kriging. Mathematical Geology 22, 239–252.
Davis, B.M., 1987. Uses and abuses of cross-validation in geostatistics. Mathematical Geology 19, 241–248.
Davis, J.C., 2002. Statistics and Data Analysis in Geology. John Wiley & Sons, New York 638 pp.
Deutsch, C.V., 2002. Geostatistical Reservoir Modeling. Oxford University Press, New York 376 pp.
Deutsch, C.V., Journel, A.G., 1998. GSLIB: Geostatistical Software Library and User's Guide second ed. Oxford University Press, New York 350 pp.
Dirks, K.N., Hay, J.E., Stow, C.D., Harris, D., 1998. High-resolution studies of rainfall on Norfolk island part II: interpolation of rainfall data. Journal of Hydrology 208, 187–193.
Dubrule, O., 1984. Comparing splines and kriging. Computers & Geosciences 10, 327–338.
Falivene, O., Cabrera, L., Sáez, A., 2007a. Optimum and robust 3D facies interpolation strategies in a heterogeneous coal zone (Tertiary As Pontes basin, NW Spain). International Journal of Coal Geology 71, 185–208.
Falivene, O., Cabrera, L., Muñoz, J.A., Arbués, P., Fernández, O., Sáez, A., 2007b. Statistical grid-based facies reconstruction and modelling for sedimentary bodies. Alluvial-palustrine and turbiditic examples. Geologica Acta 5, 199–230.
Ferrús, B., 1998. Análisis de cuenca y relaciones tectónica-sedimentación en la cuenca de As Pontes (Galícia). Basin analysis and tectono-sedimentary relationships in the As Pontes basin (Galicia). Unpublished PhD Dissertation, University of Barcelona, Barcelona (Spain), 351 pp.
Gallichand, J., Marcotte, D., 1993. Mapping clay content for subsurface drainage in the Nile Delta. Geoderma 58, 165–179.
Goovaerts, P., 2000. Geostatistical approaches for incorporating elevation into the spatial interpolation of rainfall. Journal of Hydrology 228, 113–129.
Herzfeld, U.C., Eriksson, M.G., Holmund, P., 1993. On the Influence of kriging parameters on the cartographic output – a study in mapping subglacial topography. Mathematical Geology 25, 881–900.
Isaaks, E.J., Srivastava, R.M., 1989. An introduction to Applied Geostatistics. Oxford University Press, New York 561 pp.
Jones, T.J., Hamilton, D.E., Johnson, C.R., 1986. Contouring Geologic Surfaces with the Computer. Van Nostrand Reinhold, New York 314 pp.
Journel, A., Kyriakidis, P.C., Mao, S., 2000. Correcting the smoothing effect of estimators: a spectral postprocessor. Mathematical Geology 32, 787–813.
Journel, A.G., Huijbregts, C.J., 1978. Mining Geostatistics. Academic Press, New York 600 pp.
Journel, A.G., Rossi, M., 1989. When do we need a trend in kriging? Mathematical Geology 21, 715–739.
Kane, V.E., Begovich, C.L., Butz, T.R., Myers, D.E., 1982. Interpretation of regional geochemistry using optimal interpolation parameters. Computers & Geosciences 8, 117–135.
Kupfersberger, H., Deutsch, C.V., 1999. Methodology for integrating analog geologic data in 3-D variogram modeling. American Association of Petroleum Geologists Bulletin 83, 1262–1278.
Lebel, T., Bastin, G., Obled, C., Creutin, J.D., 1987. On the accuracy of rainfall estimation: a case study. Water Resources Research 23, 2123–2134.
Lu, G.Y., Wong, D.W., 2008. An adaptive inverse-distance weighting spatial interpolation technique. Computers & Geosciences 34, 1044–1055.
Matheron, G., 1963. Principles of geostatistics. Economic Geology 58, 1246–1266.
Mitasova, H., Mitas, L., 1993. Interpolation by regularized spline with tension: I. Theory and implementation. Mathematical Geology 25, 641–655.
Morrison, J.L., 1974. Observed statistical trends in various interpolation algorithms useful for first stage interpolation. The Canadian Cartographer 11, 142–159.
Moyeed, R.A., Papritz, A., 2002. An empirical comparison of kriging methods for nonlinear spatial point prediction. Mathematical Geology 34, 365–386.
Okubo, C.H., Schultz, R.A., Stefanelli, G.S., 2004. Gridding Mars orbiter laser altimeter data with GMT: effects of pixel size and interpolation methods on DEM integrity. Computers & Geosciences 30, 59–72.
Olea, R., Pawlowsky, V., 1996. Compensating for estimation smoothing in kriging. Mathematical Geology 28, 407–417.
Pebesma, E.J., Wesseling, C.G., 1998. GSTAT: a program for geostatistical modelling, prediction and simulation. Computers & Geosciences 24, 17–31.
Sáez, A., Cabrera, L., 2002. Sedimentological and paleohydrological responses to tectonics and climate in a small, closed, lacustrine system: Oligocene As Pontes Basin (Spain). Sedimentology 49, 1073–1094.
Sáez, A., Inglès, M., Cabrera, L., de las Heras, A., 2003. Tectonic-palaeoenvironmental forcing of clay-mineral assemblages in nonmarine settings: the Oligocene–Miocene As Pontes Basin (Spain). Sedimentary Geology 159, 305–324.
Santanach, P., Ferrús, B., Cabrera, L., Sáez, A., 2005. Origin of a restraining bend in an evolving strike-slip system: the Cenozoic As Pontes basin (NW Spain). Geologica Acta 3, 225–239.
Teegavarapu, R.S.V., Chandramouli, V., 2005. Improved weighting methods, deterministic and stochastic data-driven models for estimation of missing precipitation records. Journal of Hydrology 312, 191–206.
Weber, D.D., Englund, E.J., 1992. Evaluation and comparison of spatial interpolators. Mathematical Geology 24, 381–391.
Weber, D.D., Englund, E.J., 1994. Evaluation and comparison of spatial interpolators II. Mathematical Geology 26, 589–603.
Zimmerman, D., Pavlik, C., Ruggles, A., Armstrong, P., 1999. An experimental comparison of ordinary and universal kriging and inverse distance weighting. Mathematical Geology 31, 375–390.