

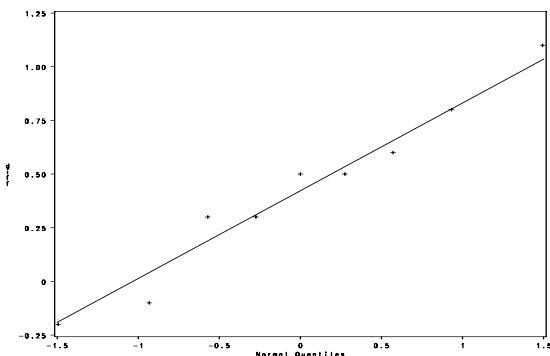
Midterm 1 Practice Problems

1. Review the fundamental principles in experimental design and their advantages?
2. Review z -test, t -test, independent two samples t -test, paired two sample t test.
3. Review type I error, type II error and power in testing hypotheses.
4. The data shown below are measurements of the amount of wear of the soles of shoes worn by 9 boys. The shoe soles were made of two different synthetic materials, A and B . Each boy wore a special pair of shoes, the sole of one shoe having been made with A and the sole of the other with B . The decision as to whether the left or the right sole was made with A or B was determined by the flip of a coin.

boy	1	2	3	4	5	6	7	8	9
A	13.2(l)	8.2(l)	10.9(r)	14.3(l)	10.7(r)	6.6(l)	9.5(r)	10.8(l)	8.8(r)
B	14.0(r)	8.8(r)	11.2(l)	14.2(r)	11.8(l)	6.4(r)	9.8(l)	11.3(r)	9.3(l)
$B - A$	0.8	0.6	0.3	-0.1	1.1	-0.2	0.3	0.5	0.5

The letters, l and r, in parentheses represent left foot and right foot respectively. The sample mean and variance of the differences are 0.422 and 0.167.

- (a) Test if material B is better than A in terms of resistance to wear and report the P -value. ($\alpha = 5\%$)
- (b) What design principles have been used in this experiment? What are their advantages?
- (c) A randomization test can in fact be used. Describe the procedure to obtain the randomization reference distribution. What is the observed statistic in the randomization test? What the P -value will **approximately** be?
- (d) The QQ-plot of the differences is obtained. Comment on the validity of the normality assumption.



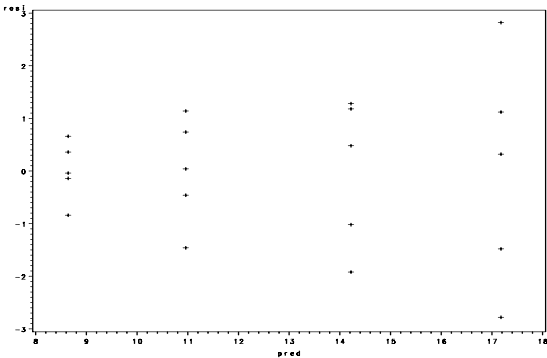
5. Four catalysts (1,2,3,4) that affect the concentration of one component in a three-component liquid solution are being investigated. The data are given below.

catalyst	concentrations				
1	12.3	15.4	13.2	15.5	14.7
2	8.5	7.8	9.3	8.6	9.0
3	11.0	12.1	11.7	10.5	9.5
4	17.5	18.3	15.7	14.4	20.0

The ANOVA table is reported as follows

Source	DF	Squares	Mean Square	F Value	Pr > F
Model	3	209.4100000	69.8033333	34.15	<.0001
Error	16	32.7000000	2.0437500		
C.Total	19	242.1100000			

- Test if the catalysts have different effects on the concentration of the component. State the hypotheses and report your conclusion.(Use $\alpha = 5\%$)
- A residual plot is generated to check model assumptions.

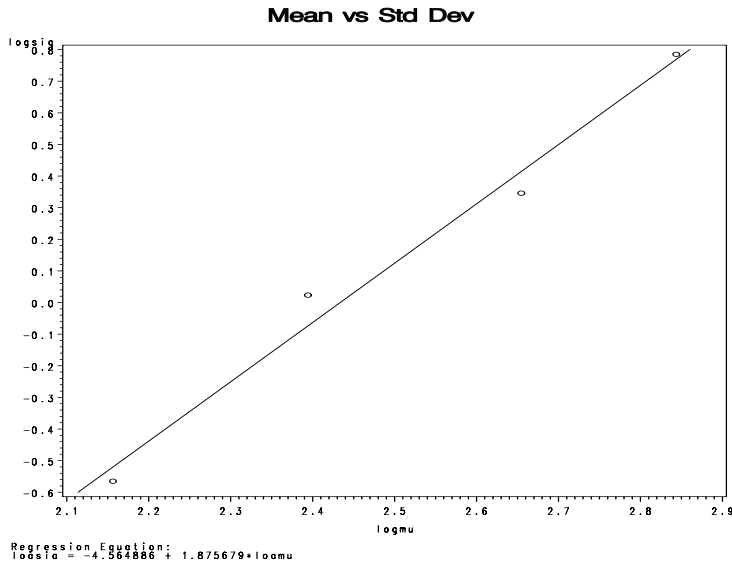


which assumption appears to be violated? Discuss its impact on a).

c) The summary statistics are given in the following,

Level of	-----resp-----		
trt	N	Mean	Std Dev
1	5	14.2200000	1.41315250
2	5	8.6400000	0.56833089
3	5	10.9600000	1.02371871
4	5	17.1800000	2.19248717

Log(Std Dev) is plotted against log(Mean), and a simple regression line is also fitted and reported.



Based on the plot, what transformation you would use to stabilize the variances?

The transformed data is re-analyzed. Suppose the following three contrasts,

$$\Gamma_1 = \mu_1 - \mu_2, \Gamma_2 = \mu_3 - \mu_4, \Gamma_3 = \mu_1 + \mu_2 - \mu_3 - \mu_4$$

are of interest.

d) Show that the three contrasts form a complete set.

e) The ANOVA table and the results of testing Γ_1 and Γ_2 are given below. Test $H_0: \Gamma_3 = 0$ with $\alpha = 5\%$.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	0.00946280	0.00315427	49.68	<.0001
Error	16	0.00101593	0.00006350		
Cored Total	19	0.01047874			

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Gamma_1	1	0.00511811	0.00511811	80.61	<.0001
Gamma_2	1	0.00270927	0.00270927	42.67	<.0001

6. An experiment was conducted to study the effect of quality control level (1=low, 2=moderate and 3=high) on the productivity improvement of a manufacturing line. The experiment was completely randomized. There are 6 observations for each level. The sample means are

$$\bar{y}_1 = 6.983, \bar{y}_2 = 8.05, \bar{y}_3 = 9.20.$$

And the corrected total sum of squares (SST) is 26.091.

- a) Construct the ANOVA table.
- b) Is there any difference between the quality control levels? State the hypotheses and use $\alpha = 5\%$.
- c) Use Tukey's procedure and Bonferroni's method to perform pairwise comparison. Report the critical differences and results.
- d) Compare the power and conservativeness of the two methods above.
- e) An analyst used orthogonal polynomial contrasts to study the relationship between quality control level and productivity improvement as follows

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contrast "linear" level -1 0 1;
contrast "quadratic" level 1 -2 1;
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Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
linear	1	14.74083333	14.74083333	19.49	0.0005
quadratic	1	0.00694444	0.00694444	0.01	0.9249

What appears to be the nature of the relationship?

Remark: This handout is to show you the type of problems you will have in exam. It is not meant to be comprehensive.