Stat514S05 Midterm II (Total 20 Points)

0. Exam time: one hour and 15 minutes.

1. Must show work to get credits.

2. Hand in both your exam and answer sheets.

1. Five machines (α, β, γ, δ, ϵ) and five operators (1, 2, 3, 4, 5) are employed in a study to compare the effects of five different assembly methods (A, B, C, D, E) on production throughput. Due to time constraints, only five runs can be conducted everyday. The following design is used for the experiment and the experiment outcome is also included.

	Operator				
Day	1	2	3	4	5
1	$A\alpha = 10.2$	$B\beta = 10.5$	$C\gamma = 8.2$	$D\delta = 14.1$	$E\epsilon = 13.2$
2	$B\gamma = 9.2$	$C\delta = 11.2$	$D\epsilon = 13.1$	$E\alpha = 11.2$	$A\beta = 9.9$
3	$C\epsilon = 9.6$	$D\alpha = 13.0$	$E\beta = 10.8$	$A\gamma = 7.3$	$B\delta = 12.9$
4	$D\beta = 12.0$	$E\gamma = 10.0$	$A\delta = 11.1$	$B\epsilon = 11.6$	$C\alpha = 10.0$
5	$E\delta = 12.3$	$A\epsilon = 11.0$	$B\alpha = 11.1$	$C\beta = 8.5$	$D\gamma = 10.0$

The grand mean $\bar{y}_{....} = 10.88$, and the level means for the five methods are

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A: 9.90   B: 11.06  C: 9.50  D: 12.44  E: 11.50
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a)(2) What kind of design is used for the experiment. Describe its major advantages.
b)(2) Calculate the estimates of the treatment effects.
c)(2) Part of the ANOVA table from SAS is given below.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	16	65.74800000	4.10925000	20.14	<.0001
Error	8	1.63200000	0.20400000		
CoTotal	24	67.38000000			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
day	4	1.25200000	0.31300000	1.53	0.2806
method	4	*********	*********	*****	*****
operator	4	1.67200000	0.41800000	2.05	0.1800
machine	4	34.24800000	8.56200000	41.97	<.0001

Test if the methods are different from each other in terms of their effects on the throughput. State the hypotheses, obtain the test statistic and draw your conclusion ($\alpha = 5\%$).
d)(2) Suppose Tukey's method is used for treatment pairwise comparison. Calculate the CD using $\alpha = 5\%$.

e)(2) If the machines were not considered as blocks and were not included in the ANOVA model, what is its impact on testing the methods?

2. The percentage of hardwood concentration (HC) in raw pulp and the vat pressure are being investigated for their effects on the strength of paper. Three levels of hardwood concentration and three levels of pressure are selected. A factorial experiment with two replicates is conducted, and the following data are obtained:

	Pressure		
HC Percentage	400	500	3: 600
2	24.9, 26.7, 23.2	27.6, 29.3, 26.3	54.3, 52.5, 55.6
4	31.5, 28.8, 25.6	37.0, 41.4, 44.0	34.0, 35.4, 42.8
6	20.4, 25.1, 26.1	35.0, 38.0, 27.0	49.6, 43.6, 53.0

Some summary statistics are given below.

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grand mean: 35.51

HC Percent      MEAN   |    Pressure     MEAN
1              35.60   |        1       25.81
2              35.61   |        2       33.96
3              35.31   |        3       46.76

HC Percent   Pressure         MEAN
1            1                24.93
1            2                27.73
1            3                54.13
2            1                28.63
2            2                40.80
2            3                37.40
3            1                23.87
3            2                33.33
3            3                48.73
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Suppose the following statistical model is considered,

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk} \tag{1}$$

where $\tau_i (i = 1, 2, 3)$, $\beta_j (j = 1, 2, 3)$ and $(\tau\beta)_{ij}$ are the main effects of HC percentage, the main effects of pressure, and their interactions, respectively, which satisfy the additional constraints given in the lecture notes.

a)(2) Calculate the estimates of $\beta_1, \beta_2, \beta_3$ and $(\tau\beta)_{22}$.

b)(2) Calculate the sum of squares due to the main effects of pressure.

c)(2) Part of the ANOVA table from SAS is given below.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	2739.531852	342.441481	26.66	<.0001
Error	18	231.206667	12.844815		
Cor.Total	26	2970.738519			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
percent	2	0.520741	0.260370	0.02	0.9800
pressure	*	*******	********	*****	******
percent*pressure	*	*******	********	*****	******

Test if the interaction between HC percentage and pressure is significant ($\alpha = 5\%$).

d)(2) Use the Bonferroni method to compare the following treatments (i.e., level combinations of the two factors): (2,1), (2,2), (2,3) and (3,2), pairwisely. Calculate the critical difference and report your result using $\alpha = 6\%$.

e)(2) An incomplete interaction plot for HC percent and pressure is given below. Please complete the plot and interpret the interaction. Note that sufficient information is already provided in the above.