

Stat514 Midterm II

1. Four different washing solutions (1, 2, 3, 4) are being compared to study their effectiveness in retarding bacteria growth in some milk containers. The experiment is done in a laboratory, and only three trials (runs) can be conducted on any day. It is known that days could represent a potential source of variability. The experimenter uses the following design, and the data are also shown here.

Solution	Day				$y_{i.}$
	1	2	3	4	
1	3.11	3.70	3.60		10.41
2		3.20	3.11	3.22	9.53
3	2.95		3.52	3.54	10.01
4	2.82	2.97		3.18	8.97
$y_{.j}$	8.88	9.87	10.23	9.94	

- a)(3) What kind of design is used by the experimenter? Determine the design parameters.
 b)(2) The ANOVA table generated from SAS is given below.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	0.81597500	0.13599583	10.26	0.0109
Error	5	0.06629167	0.01325833		
CoTotal	11	0.88226667			

Source	DF	Type III SS	Mean Square	F Value	Pr > F
day	3	0.42984167	0.14328056	10.81	0.0126
solution	3	0.47057500	0.15685833	11.83	0.0104

Test whether the four solutions are different in retarding bacteria growth.

- c)(2) The least square means for Solutions 2, 3 and 4 are given below.

solution	resp LSMEAN	LSMEAN Number
1	*****	1
2	3.06208333	2
3	3.36583333	3
4	3.02083333	4

Calculate the least squares mean (adjusted mean) for Solution 1.

- d)(2) What is the standard error for the least squares means?
 e)(3) Suppose the Bonferroni method is used to compare the four solutions. Calculate the critical difference (CD) at $\alpha = 6\%$.

2. An experiment is conducted to study the effects of loading frequencies (Frequency) and environmental conditions (Environment) on fatigue crack growth at a constant 22 MPa stress for a particular material. The data from this experiment are shown below (the response is crack growth rate):

Frequency	Environment		
	1: Air	2: H ₂ O	3: SaltH ₂ O
1	2.29, 2.47, 2.12	2.86, 3.03, 2.73	4.93, 4.75, 5.06
2	3.15, 2.88, 2.56	4.00, 4.44, 4.70	3.10, 3.24, 3.98
3	2.24, 2.71, 2.81	4.00, 4.30, 3.20	4.86, 4.26, 5.20

Some summary statistics are given below.

grand mean: 3.551

frequency	MEAN	environment	MEAN
1	3.360	1	2.581
2	3.561	2	3.696
3	3.731	3	4.376

frequency	environment	MEAN
1	1	2.293
1	2	2.873
1	3	4.913
2	1	2.863
2	2	4.380
2	3	3.440
3	1	2.587
3	2	3.833
3	3	4.773

Suppose the following statistical model is considered,

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk} \quad (1)$$

where $\tau_i (i = 1, 2, 3)$, $\beta_j (j = 1, 2, 3)$ and $(\tau\beta)_{ij}$ are the main effects of frequency, the main effects of environment and their interactions, respectively, which satisfy the additional constraints given in the lecture notes.

a)(3) Calculate the estimates of τ_1, τ_2, τ_3 and $(\tau\beta)_{23}$.

b)(2) Calculate the sum of squares due to frequency.

c)(4) The ANOVA table from SAS is given below. Notice that some quantities are removed.

Dependent Variable: crack

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	22.719	2.840	22.11	<.0001
Error	18	2.312	0.128		
Coed Total	26	25.031			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
frequency	*	*****	*****	****	*****
environment	2	14.773	7.387	57.51	<.0001
frequency*environment	*	*****	*****	****	*****

Test if the main effects of frequency are significant and if the interactions of frequency and environment are significant? (use $\alpha = 5\%$).

d)(3) Calculate the critical difference (CD) for the treatment pairwise comparison using Tukey's method. Report the results for the following pairs: (1,1) versus (1,3); (2,2) versus (3,2).

e)(2) The interaction plot for frequency and environment is generated and included below. Based on the information in the plot and your answers in c), explain the overall impact of frequency on fatigue crack growth.

