Stat514 Midterm II

1. The effect of five different ingredients (A, B, C, D, E) on the reaction time of a chemical process is being studied. Each batch of new material is only large enough to permit five runs to be made. Furthermore, each run requires approximately 1.5 hours, so only five runs can be made in one day. The experiment layout and results are given below.

	Day				
Batch	1	2	3	4	5
1	$A = 6$	$B = 6$	$D = 1$	$C = 8$	$E = 5$
2	$C = 10$	$E = 2$	$A = 8$	$D = 5$	$B = 11$
3	$B = 2$	$A = 8$	$C = 10$	$E = 2$	$D = 7$
4	$D = 1$	$C = 4$	$E = 3$	$B = 4$	$A = 8$
5	$E = 2$	$D = 1$	$B = 3$	$A = 9$	$C = 10$

The grand mean $\bar{y}_{...} = 5.44$. And the level means for batch, day and ingredient are

Level of batch	N	Mean
1	5	5.20
2	5	7.20
3	5	5.80
4	5	4.00
5	5	5.00

Level of day	N	Mean
1	5	4.20
2	5	4.20
3	5	5.00
4	5	5.60
5	5	8.20

Level of Ingredient	N	Mean
1	5	7.80
2	5	5.20
3	5	8.40
4	5	3.00
5	5	2.80

a)(2) What design is empolyed for the experiment. Describe its major advantages.
b)(3) Write down a proper statistical model for the design. What are the estimates of the treatment effects?
c)(3) Part of the SAS output for ANOVA is given below

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	12	218.880	18.2400	5.57	0.0029
Error	12	39.280	3.2733		
Co. Total	24	258.160			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
batch	4	27.760	6.940	2.12	0.1410
day	4	54.560	13.640	4.17	0.0241
ingredient	*	******	******	****	******

Test if the five gredients have different effect on the chemical reaction time. State the hypotheses, obtain the test statistic and draw your conclusion (use $\alpha = 5\%$).
d)(3) Use Tukey's method for treatment pairwise comparison. Calculate the critical difference and draw your conclusions.
e)(2) Assume that five operators are employed to conduct the experiment and it is known that the operators can influence the experimental results. Derive an experimental plan that can be used to

study the five ingredients using five batches, five operators and in five days. Some useful squares are given below.

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A B C D E   A B C D E   A B C D E   A B C D E
B C D E A   C D E A B   E A B C D   D E A B C
C D E A B   E A B C D   D E A B C   B C D E A
E A B C D   D E A B C   B C D E A   C D E A B
D E A B C   B C D E A   C D E A B   E A B C D
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2. An engineer suspects that the surface finish of a metal part is influenced by the feed rate and the depth of cut. He selects three feed rates and four depths of cut. He then conducts a factorial experiment and obtains the following data.

	Depth of Cut (in)			
Feed Rate (in/min)	1	2	3	4
1	74, 64, 60	79, 68, 73	82, 88, 92	99, 104, 96
2	92, 86, 88	98, 104, 88	99, 108, 95	104, 110, 99
3	99, 98, 102	104, 99, 95	108, 110, 99	114, 111, 107

Some summary statistics are give below.

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Grand mean: 94.333

feed        mean      |     depth   mean
1          81.583     |     1       84.778
2          97.583     |     2       89.778
3         103.833     |     3       97.889
                      |     4      104.889
-----------------------------------------------
feed    depth       mean
1       1           66.000
1       2           73.333
1       3           87.333
1       4           99.667
2       1           88.667
2       2           96.667
2       3          100.667
2       4          104.333
3       1           99.667
3       2           99.333
3       3          105.667
3       4          110.667
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Suppose the following statistical model is used to fit the data.

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk}, k = 1, 2, 3$$

where τ_i $(i = 1, 2, 3)$ and β_j $(j = 1, 2, 3, 4)$ are the effects of feed rate and cut depth, and $(\tau\beta)_{ij}$ are their interactions.

a)(2) What are the constraints τ_i, β_j and $(\tau\beta)_{ij}$ need satisfy?

b)(3) What are the estimates of τ_2 and $(\tau\beta)_{22}$?

c)(3) The ANOVA of the data was done in SAS and the output is as follows.

Dependent Variable: finish

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	5842.667	531.151515	18.49	<.0001
Error	24	689.333	28.722222		
Co Total	35	6532.000			

ource	DF	Type I SS	Mean Square	F Value	Pr > F
feed	2	3160.500	1580.250	55.02	<.0001
depth	3	2125.111	708.370	24.66	<.0001
feed*depth	6	557.056	92.843	3.23	0.0180

Test if the main effects and interactions are significant (use $\alpha = 5\%$)

d)(3) Use the Bonferroni method to compare the following treatments (i.e. the lelve combniations of speed and depth): (2,3), (2,4), (3,3) and (3,4) (Use $\alpha = 4\%$). Calcuate the critical difference and report you results.

e)(1) An interaction plot for feed rate and cut depth is included. Interpret the interaction between feed rate and cut depth.