Stat514S05 Midterm I (Total 40 Points)

0. Exam time: one hour and 15 minutes.

1. Use $\alpha = 5\%$ throughout the exam.

2. Must show work to get credits.

3. Hand in both your exam and answer sheets.

1. An experiment was run to determine whether four specific temperatures affect the production of a certain type of chemical compound. The experiment led to the following data. The response is the amount of the chemical compound produced over a period of time.

Temperature	Response					Mean	St.D.
100	8.71	10.47	9.62	11.55	10.25	10.12	1.05
125	24.12	25.23	26.08	20.30	21.15	23.38	2.54
150	30.37	31.14	35.62	30.19	27.03	30.87	3.09
175	28.24	24.15	22.02	26.46	27.44	25.66	2.55

a)(2) Generate the plot of log St.D. ($\log s_i$) versus log Mean ($\log \bar{y}_{i.}$) by hand.
b)(3) Based on your plot, is the constant variance assumption for ANOVA valid? What remedy you can recommend? Derive the remedy explicitly.

The modified data are given below.

Temperature	Transformed Response					Mean	St.D.
100	2.16	2.35	2.26	2.45	2.33	2.310	0.1079
125	3.18	3.23	3.26	3.01	3.05	3.146	0.1106
150	3.41	3.44	3.57	3.41	3.30	3.426	0.0966
175	3.34	3.18	3.09	3.28	3.31	3.240	0.1032

c)(5) It is known that SST=3.839. Construct the ANOVA table for testing if the temperatures have different effects on the transformed response.
d)(2) What is the estimate for τ_1 (the treatment effect at temperature 100)?
e)(2) Let μ_1, μ_2, μ_3 and μ_4 be the treatment means at temperatures 100, 125, 150 and 175, respectively, and let $L = \mu_1 - 2\mu_2 + \mu_3$. What is the estimate for L_1?
f)(3) Test $H_0 : L_1 = 0 \,\text{vs}\, H_1 : L_1 \neq 0$.
g)(3) One decides to use the contrasts based on orthogonal polynomials to further model the functional relationship between temperature and the transformed response. Part of the SAS code and output is given below.

```
contrast 'linear'    temperature -3 -1  1 3;
contrast 'quadratic' temperature  1 -1 -1 1;
contrast 'cubic'     temperature -1  3 -3 1;
-----------------------------------------------------------------
```

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
linear	*	*********	********	******	<.0001
quadratic	1	1.30560500	1.30560500	119.07	<.0001
cubic	1	0.00202500	0.00202500	0.18	0.6731

Fill in the DF, Contrast SS, Mean Square and F Value for the linear contrast.

2) The effective life of insulating fluids at an accelerated load of 35kV is being studied. Test data have been obtained for four types of fluids. The results were as follows.

Type	Life(in hour) at 35kV load					Mean
1	17.6	18.9	16.3	17.4	20.1	18.06
2	16.4	14.9	18.1	16.6	19.0	17.00
3	21.9	24.1	19.9	19.0	21.0	21.18
4	19.3	21.1	16.9	17.5	18.3	18.62

The ANOVA table obtained from SAS is given below.

Source	DF	Sum of Squares	Mean Square	F Value
Model	3	47.27750000	15.75916667	5.58
Error	16	45.14800000	2.82175000	
Corrected Total	19	92.42550000		

a)(2) Test if there exist differences between the types of fluids.

b)(2) What is the estimate for population variance σ^2?

c)(4) Use Scheffe's method to construct simultaneous confidence intervals for $\Gamma_1 = \mu_1 - 2\mu_2 + \mu_3$ and $\Gamma_2 = \mu_2 - 2\mu_3 + \mu_4$.

d)(2) Use your answer in c) to test $H_0 : \Gamma_1 = \Gamma_2 = 0$.

e)(4) Use the Bonferroni method to perform pairwise comparison between the treatment means, that is, to calculate the CD and report the results. Some critical values for $t(16)$ are given below.

f)(4) Use Tukey's method to perform pairwise comparison between the treatment means, that is, to calculate the CD and report the results.

g)(2) Are the results in e) and f) consistent? Which method you prefer? Give your reason.

$t_{0.05/1}(16)$ 1.746	$t_{0.05/2}(16)$ 2.120	$t_{0.05/3}(16)$ 2.328	$t_{0.05/4}(16)$ 2.473	$t_{0.05/5}(16)$ 2.583	$t_{0.05/6}(16)$ 2.673	$t_{0.05/7}(16)$ 2.748
$t_{0.05/8}(16)$ 2.813	$t_{0.05/9}(16)$ 2.870	$t_{0.05/10}(16)$ 2.921	$t_{0.05/11}(16)$ 2.967	$t_{0.05/12}(16)$ 3.008	$t_{0.05/13}(16)$ 3.047	$t_{0.05/14}(16)$ 3.082