Stat514 Midterm I (Total 40 Points)

1. An economist compiles data on productivity improvement in percentage last year for a sample of companies that manufacture electronic computing equipments. The companies are classified according to the level of their expenditures for research and development (R&D) in the past three years (low=1, moderate=2, high=3). Five companies are randomly selected for each level and their averages and standard deviations are

Level of		improvement	
R&D	N	Mean	Std Dev
1	5	12.3800000	0.78396428
2	5	8.0260000	1.07195149
3	5	13.2060000	1.27852650

a)(5) It is given that SST=91.047. Construct the ANOVA table.

b)(2) Test if the R&D expenditure level affects the productivity improvement. State the hypotheses and use $\alpha = 5\%$.

c)(3) Suppose the economist is interested in the following two contrasts:

$$\Gamma_1 = \mu_1 - \mu_2,\ \Gamma_2 = \mu_2 - \mu_3.$$

Use the Bonferroni method to construct confidence intervals for Γ_1 and Γ_2 with overall confidence level at least 90%.

d)(3) Re-do (c) using the Scheffe method.

e)(3) Based on the results in c) and d), test

$$H_0:\ \Gamma_1 = 0 \ \text{ and } \ \Gamma_2 = 0,$$

at $\alpha = 10\%$. Which method should be preferred here? why?

The economist further uses the orthogonal polynomial contrasts to study the relationship between the productivity improvement and the R&D expenditure level. The SAS output follows.

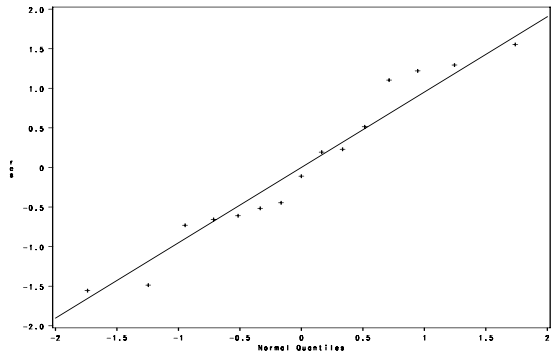
Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
linear	1	1.70569000	1.70569000	1.51	0.2433
quadratic	1	75.74763000	75.74763000	66.87	<.0001

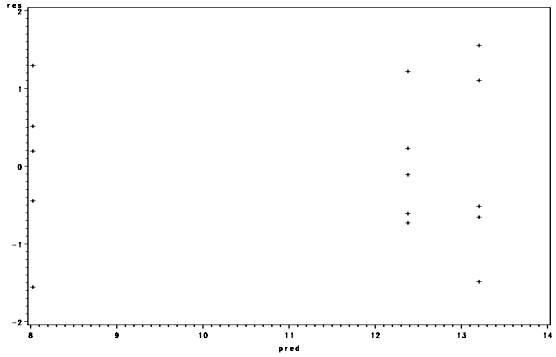
f)(2) Write down the linear and quadratic contrasts.

g)(2) What is your conclusion about the relationship between the productivity improvement and

the R&D expenditure level based on the SAS output above?

h)(2) The following two plots are generated by SAS. Based on the plots, comment on the validity of the model assumptions.





2. In an experiment, the amount of radon released in shower was investigated. Radon-enriched water was used, and five different orifice diameters were tested in shower heads. The data from the experiment are shown in the following table (the response is the percentage of radon released).

Orifice Diameter	Radon Release %				Mean	St.D.
0.40	87	88	89	93	89.25	2.62995564
0.60	74	73	76	77	75.00	1.82574186
0.80	69	71	70	72	70.50	1.29099445
1.00	76	72	74	74	74.00	1.63299316
1.20	89	92	84	89	88.50	3.31662479

The ANOVA table obtained from SAS is given below.

Source	DF	Sum of Squares	Mean Square	F Value
Model	4	1230.200	307.550000	60.11
Error	15	76.750	5.116667	
Corr Total	19	1306.950		

a)(2) Test if the diameter of orifice affects the mean percentage of radon released? Use $\alpha = 5\%$.
b)(2) What is the estimate of σ^2?
c)(5) Duncan's multiple range test and Tukey's studentized range test are used for pairwise comparison. The results are shown below (left:Duncan; right:Tukey; separated by ||).

Duncan Grouping	Mean	N	trt		Tukey Grouping	Mean	N	trt
A	89.250	4	1	||	A	89.250	4	1
A	88.500	4	5	||	A	88.500	4	5
				||				
B	75.000	4	2	||	B	75.000	4	2
B	74.000	4	4	||	B	74.000	4	4
				||	B			
C	70.500	4	3	||	B	70.500	4	3

Remark: Duncan's method was not covered in this semester so it is not required.

These two methods can lead to different answers. Use the comparison between treatment 3 and treatment 4 as an example to explain the inconsistency between these two methods, that is, calculate the critical differences and comment on their overall error rates and testing powers.

Notice that the amount of released radon changes when the size of orifice varies from 0.40 to

1.20 in diameter. An analyst wants to study the functional relationship between the response and the diameter. She obtains the complete set of orthogonal contrasts from Table X in Montgomery:

$$\begin{array}{lrrrrr} C1: & -2 & -1 & 0 & 1 & 2 \\ C2: & 2 & -1 & -2 & -1 & 2 \\ C3: & -1 & 2 & 0 & -2 & 1 \\ C4: & 1 & -4 & 6 & -4 & 1 \end{array}$$

The contrast sum of squares for $C1$, $C3$ and $C4$ and their testing results are given below.

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
C1	1	2.500000	2.500000	0.49	0.4952
C2	*	********	********	****	******
C3	1	0.625000	0.625000	0.12	0.7316
C4	1	1.289286	1.289286	0.25	0.6230

d)(3) Obtain the estimate of $C1$ and test if there exists a significant linear trend in the functional relationship?($\alpha = 5\%$).

e)(4) Note that the contrast SS, Mean Square, F Value and Pr $>$ F for $C2$ are missing. Recover these values and test if $C2$ is significant ($\alpha = 5\%$) (only an upper bound for Pr $>$ F is needed).

f)(2) What is your conclusion regarding the functional relationship between the amount of radon released and the diameter of orifice?