Stat514F05 Midterm II Solution

1.

a) For Γ_1, its estimate is $C_1 = \hat{\mu}_1 - \hat{\mu}_3 = 26.41 - 19.60 = 6.81$, and the standard error is

$$\mathrm{S.E.}_1 = \sqrt{\mathrm{MSE}(\frac{1}{5} + \frac{1}{5})}\sqrt{5.27 * \frac{2}{5}} = 1.452.$$

Because the two contrasts need to be considered simultaneously, that is, $m = 2$, the t critical value is $t_{.05/2*2}(16) = 2.473$. The critical difference is

$$\mathrm{CD}_1 = \mathrm{S.E.}_1 * \mathrm{t}_{.05/2*2}(16) = 1.452 * 2.473 = 3.591.$$

Hence the confidence interval for Γ_1 is

$$\mathrm{CI}_1 : (\mathrm{C}_1 - \mathrm{CD}_1, \mathrm{C}_1 + \mathrm{CD}_1) = (6.81 - 3.591, 6.81 + 3.591).$$

For Γ_2, similarly, $C_2 = \hat{\mu}_1 + \hat{\mu}_2 - 2\hat{\mu}_4 = -3.32$,

$$\mathrm{S.E.}_2 = \sqrt{\mathrm{MSE}(\frac{1}{5} + \frac{1}{5} + \frac{4}{5})} = 2.544,$$

and $CD_2 = 6.29$. Hence, the confidence interval for Γ_2 is

$$\mathrm{CI}_2 : (-3.32 - 6.29, -3.32 + 6.29)$$

b) Because CI_1 does not contain zero, reject H_0.

c) Applying Scheffe's method, we have

$$\mathrm{CD} = \sqrt{(\mathrm{a}-1)\mathrm{F}_{\alpha,\mathrm{a}-1,\mathrm{N}-\mathrm{a}}}\mathrm{S.E.} = \sqrt{(\mathrm{a}-1)\mathrm{F}_{\alpha,\mathrm{a}-1,\mathrm{N}-\mathrm{a}}}\sqrt{\mathrm{MSE}(\frac{1}{5} + \frac{1}{5})}$$

$$= \sqrt{(4-1) * 3.25}\sqrt{5.27 * (\frac{1}{5} + \frac{1}{5})} = 4.53.$$

Comparing the differences between the treatment means with CD, we conclude that (A, C) and (D, C) are significant.

d)Using Tukey's method,

$$\mathrm{CD} = \frac{\mathrm{q}_\alpha(\mathrm{a}, \mathrm{N}-\mathrm{a})}{\sqrt{2}}\sqrt{\mathrm{MSE}(\frac{1}{5} + \frac{1}{5})} = 4.16.$$

Similarly, we claim that (A, C) and (D, C) are significant.

e) Both methods control the type I error. Scheffe's approach is aimed at simultaneously testing all the contrasts of which pairwise comparisons form a subset. Hence it is conservative, especially when the number of pairs is small. Tukey's method is designed for pairwise comparisons only. It should be preferred here.

2.

a) Blocking is used. Blocking helps eliminate day-to-day variation in analysis.

b)

$$\mathrm{SS_{days}} = 3 * [(5.867 - 5.358)^2 + (5.933 - 5.358)^2 + (4.867 - 5.358)^2 + (4.80 - 5.358)^2] = 3.433.$$

c) Similarly, we have $\mathrm{SS_{solution}} = 14.563$. So

$$\mathrm{SS_E} = 18.17 - 14.56 - 3.34 = 0.18,$$

and

$$F_0 = \frac{\mathrm{SS_{solution}}/(\mathrm{a}-1)}{\mathrm{SS_E}/(\mathrm{a}-1)(\mathrm{b}-1)} = \frac{14.563/2}{0.18/2*3} = 242.6.$$

Because $F_0 > F_{0.05,2,6}$, we claim the treatment effects are different.

d) Assume $y_{ijk} = \mu + \tau_i + \beta_j + \gamma\tau_i\beta_j + \epsilon_{ijk}$. The hypotheses are

$$H_0 : \gamma = 0, \text{ versus } H_a : \gamma \neq 0$$

Given $\mathrm{SS_N} = 0.0237$, $\mathrm{SS'_E} = \mathrm{SS_E} - \mathrm{SS_N} = 0; 1563$. So

$$F_0 = \frac{\mathrm{SS_N}/1}{\mathrm{SS'_E}/[(\mathrm{a}-1)(\mathrm{b}-1)-1]} = \frac{0.0237}{0.1563/5} = 0.76$$

Because $F_0 < F_{0.05,1,5}$, fail to reject H_0.

3.

a)

$$\begin{array}{cccc} B & C & D & \mathbf{A} \\ C & D & \mathbf{A} & \mathbf{B} \\ D & A & \mathbf{B} & \mathbf{C} \\ A & B & \mathbf{C} & \mathbf{D} \end{array}$$

b) Block in two directions (day and batch) and eliminate the variation caused by them.

c) The responses at A are 31, 24, 23, and 27, so

$$\bar{y}_{1..} = \frac{31 + 24 + 23 + 27}{4} = 26.25,$$

and $\hat{\tau}_1 = 26.25 - 21.5625 = 4.6875$.

d)

$$\text{SS}_{\text{trt}} = 4 * [(26.25 - 21.5625)^2 + (23 - 21.5625)^2 + (17 - 21.5625)^2 + (20 - 21.5625)^2] = 189.1875$$

e)

$$\text{CD} = \frac{\text{q}_\alpha(4, (4-1)(4-2))}{\sqrt{2}} \sqrt{\text{MSE}(\frac{1}{4} + \frac{1}{4})} = \frac{4.90}{\sqrt{2}} \sqrt{3.815 * 2} = 4.785.$$

The significant pairs are (A, D), (A, C) and (B, C).