Midterm 1 Solution

Problem 1

a) The factor is the safety program of two levels: with or without it;

b) Though all the three principles randomization, replication and blocking were followed in the experiment, the primary one is blocking. Blocking helps elimination the variation between different plants.

c) First rank the differences from the smallest to the largest. Because 9 is the second largest in a sample of 10 observations, $i = 9$. Hence,

$$\alpha_i = \frac{9 - 3/8}{10 + 1/4} = 0.84.$$

The z-value corresponding to $\alpha_i = 0.84$ is approximately 1.00. So, the point in the QQplot is (9,1).

d) Let μ be the mean difference before and after the safety program was implemented. The hypotheses are

$$H_0 : \mu = 0 \text{ versus } H_1 : \mu > 0.$$

Using the sample of differences, $\bar{x}$=5.2, $s^2 = 16.62$. So

$$t_d = \frac{\bar{x} - 0}{s/\sqrt{n}} = 5.2/\sqrt{16.62}/\sqrt{10} = 4.03.$$

Because $t_d > t_{0.05}(9) = 1.83$, reject H_0 and conclude that the safety program is effective.

e)Because the variance is known, $\sigma^2 = 16$, so we can use the z-test as follows.

$$z_o = \frac{\bar{x} - 0}{\sigma/\sqrt{n}} = 5.2/4/\sqrt{10} = 4.11.$$

Because $z_o > z_{.05} = 1.645$, reject H_0.

f) The decision rule used in e) is reject H_0 if $Z_o = \frac{\bar{X}-0}{\sigma/\sqrt{n}} > z_{.05}$. When the mean difference is 4, that is, $\mu = 4$, the type II error of the decision rule is

$$\beta = P(\text{not reject } H_0|\mu = 4) = P(z_o = \frac{\bar{X} - 0}{\sigma/\sqrt{n}} \leq z_{.05}|\mu = 4)$$

$$= P(\frac{Z - 4}{\sigma/\sqrt{10}} \leq z_{.05} - \frac{4}{\sigma/\sqrt{10}}) = P(Z \leq 1.645 - \sqrt{10}) = P(Z \leq -1.517) = 0.065.$$

Because $\beta > .05$, the sample size is not large enough. In fact, one more observation is needed.

Problem 2 a) $\bar{y}_{..} = (26.41 + 22.69 + 19.60 + 26.16)/4 = 23.715$, and

$$\mathrm{SS_{tr}} = \sum_{\mathrm{i}=1}^{4} \mathrm{n_i}(\bar{\mathrm{y}}_{\mathrm{i.}} - \bar{\mathrm{y}}_{..})^2$$

$$= 5*(26.41-23.715)^2+5*(22.69-23.715)^2+5*(19.60-23.715)^2+5*(26.16-23.715)^2 = 156.12$$

or

$$\mathrm{SS_E} = \sum_{\mathrm{i}=1}^{4}(\mathrm{n_i}-1)\mathrm{s_i^2} = 4*(2.37)^2+4*(2.23)^2+4*(2.32)^2+4*(2.26)^2 = 84.32$$

So the ANOVA table is

Source	DF	Sum of Squares	Mean Square	F Value
Between	3	156.12	52.04	9.87
Error	16	84.32	5.27	
Total	19	240.44		

Because $F_0 > F_{0.05}(4,15) = 3.06$, conclude that dosage levels affect the bioactivity of the new drug.

b)

$$\hat{\tau}_2 = \bar{y}_{2.} - \bar{y}_{..} = 22.69 - 23.715 = -1.025.$$

c) The constant variance assumption seems to hold, because the sample variances at various levels of the factor are fairly close to each other.

d)

$$\hat{\sigma}^2 = \mathrm{MSE} = 5.27.$$

e) It can be verified that the contrasts are mutually orthogonal and there are three of them, so they form a complete set of orthogonal contrasts.

f)

$$\hat{\Gamma}_2 = \hat{\mu}_2 - \hat{\mu}_3 = \bar{y}_{2.} - \bar{y}_{3.} = 22.69 - 19.60 = 3.09.$$

g) Because Γ_1, Γ_2 and Γ_3 form a complete set of orthogonal contrasts, the contrast SS of Γ_3 is

$$\mathrm{SS_{C_3}} = \mathrm{SS_{tr}} - \mathrm{SS_{C_1}} - \mathrm{SS_{C_2}}$$

$$= 156.12 - 0.156 - 23.808 = 132.16.$$

So,

$$F_0 = \frac{132.16}{5.27} = 25.08$$

and the P-value is less than 0.01.

h) Based on the results above, it appears that μ_1 and μ_4 are almost the same, and μ_2 and μ_3 are not dramatically different from each other; the middle levels are significantly different from the low and high levels.