Homework 7 Solution

Problem 1

(a) Let's look at the ANOVA table output from SAS (I replaced the line for model sum of squares by the lines for treatment SS and block SS).

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
chem	3	12.9500000	4.3166667	2.38	0.1211
bolt	4	157.0000000	39.2500000	21.61	<.0001
Error	12	21.8000000	1.8166667		
Corrected Total	19	191.7500000			

Since p-value for treatment effect is $0.1211 > 0.05$, I conclude that the mean effects of the four chemical agents on the strength of the cloth are not quite different. Also, p-value for the block effect is very small (< 0.0001), so the block effect (bolt effect) dominates the variations in the experiment.

(b) The diagnostic plots in Figure 1 are: normal probability Q-Q plot, plot of residuals versus chemical agents (treatment), plot of residuals versus bolt (block), and plot of residuals versus predicted values.

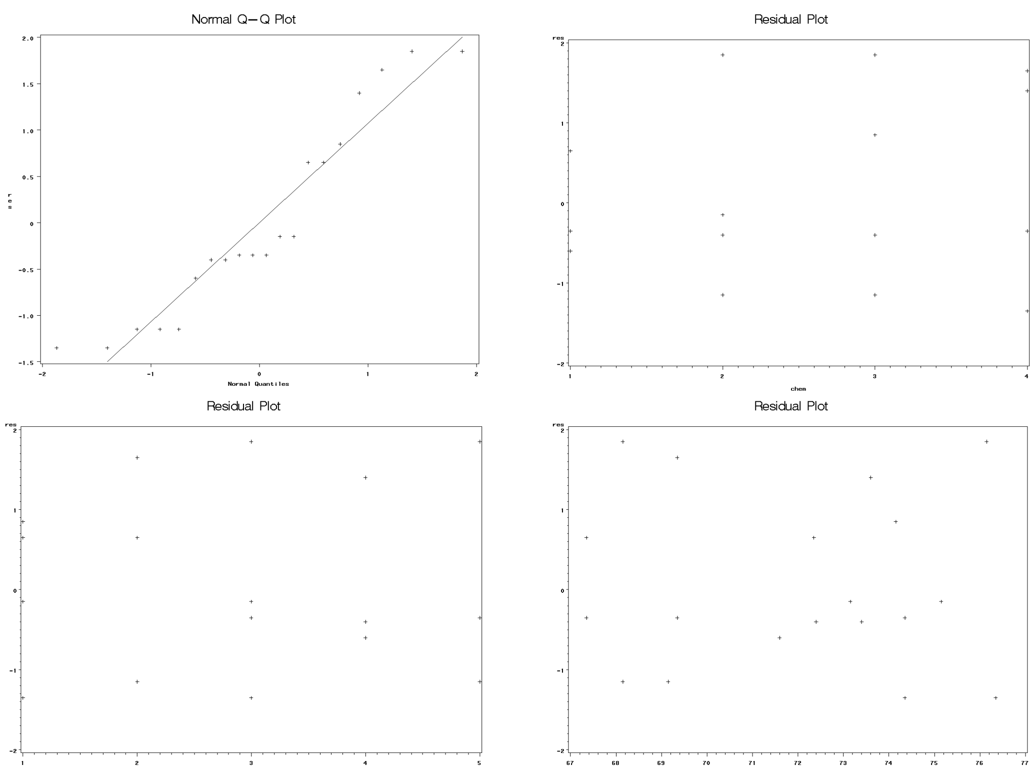

Figure 1: Diagnostic Plots

The normal Q-Q plot doesn't show any severe violation of the normality assumption. Neither of the three residual plots indicates any inequality variances or curvilinearities. And there are no potential outliers or influential points in the plots.

(c) The 95% confidence intervals for treatment means are listed in the following SAS output.

```
chem        y LSMEAN      95% Confidence Limits

1          70.600000       69.286674    71.913326
2          71.400000       70.086674    72.713326
3          72.400000       71.086674    73.713326
4          72.600000       71.286674    73.913326
```

(d) The following SAS output gives the comparison results for the Tukey and Bonferroni methods.

```
                         Tukey's Studentized Range (HSD) Test for y

NOTE: This test controls the Type I experimentwise error rate, but it generally has a higher
                               Type II error rate than REGWQ.


                         Alpha                                    0.05
                         Error Degrees of Freedom                   12
                         Error Mean Square                    1.816667
                         Critical Value of Studentized Range  4.19852
                         Minimum Significant Difference         2.5307


                 Means with the same letter are not significantly different.


                 Tukey Grouping          Mean      N    chem

                             A        72.6000      5    4
                             A
                             A        72.4000      5    3
                             A
                             A        71.4000      5    2
                             A
                             A        70.6000      5    1


                             Bonferroni (Dunn) t Tests for y

NOTE: This test controls the Type I experimentwise error rate, but it generally has a higher
                               Type II error rate than REGWQ.


                            Alpha                               0.05
                            Error Degrees of Freedom              12
                            Error Mean Square               1.816667
                            Critical Value of t              3.15268
                            Minimum Significant Difference    2.6875


                 Means with the same letter are not significantly different.


                   Bon Grouping          Mean      N    chem

                             A        72.6000      5    4
                             A
                             A        72.4000      5    3
                             A
                             A        71.4000      5    2
                             A
                             A        70.6000      5    1
```

Neither Tukey's method nor Bonferroni's method detects any differences among the four treatment groups. This is consistent with our ANOVA result in (a). ∎

Problem 2

(a) The treatment sum of squares can be calculated by

$$\begin{aligned} SS_{\text{Treatment}} &= b\sum_{i=1}^{a} \bar{y}_{i\cdot}^2 - N\bar{y}_{\cdot\cdot}^2 \\ &= 5*(5.40^2 + 5.80^2 + 10^2 + 9.80^2) - 20*7.75^2 \\ &= 92.95 \end{aligned}$$

Then the F statistic for testing the treatment effect is

$$F = \frac{SS_{\text{Treatment}}/(a-1)}{MS_E} = \frac{92.95/3}{6.275} = 4.938,$$

which is greater than $F_{0.05,3,12} = 3.49$, the 95% percentile of distribution $F_{3,12}$. Hence I conclude that there are differences among the four cooling temperatures.

(b) The critical distance for Tukey's pairwise comparisons method is

$$CD = q_{\alpha,a,(a-1)(b-1)}\sqrt{MS_E/b} = q_{0.05,4,12}\sqrt{6.275/5} = 4.705.$$

The maximum difference among the four temperature groups is $10 - 5.40 = 4.60$, which is less than the above critical difference. Hence Tukey's method detects no significant differences among the four temperature groups. As shown below, my computation is consistent with the SAS output.

```
                     Tukey's Studentized Range (HSD) Test for y

NOTE: This test controls the Type I experimentwise error rate, but it generally has a higher
                           Type II error rate than REGWQ.


                    Alpha                                   0.05
                    Error Degrees of Freedom                  12
                    Error Mean Square                      6.275
                    Critical Value of Studentized Range  4.19852
                    Minimum Significant Difference        4.7035


                 Means with the same letter are not significantly different.


                 Tukey Grouping          Mean      N    tmp

                              A        10.000      5    3
                              A
                              A         9.800      5    4
                              A
                              A         5.800      5    2
                              A
                              A         5.400      5    1
```

(c) The contrasts are

$$\begin{aligned} \mathbf{C}_1 &= (1,-1,0,0), \\ \mathbf{C}_2 &= (0,0,1,-1), \\ \mathbf{C}_3 &= (1,1,-1,-1), \end{aligned}$$

where $\mathbf{C}_1$ is used to test the difference between temperatures 5° and 10°, $\mathbf{C}_2$ to test the difference between temperatures 15° and 20°, and $\mathbf{C}_3$ to test the difference between lower temperatures (5°, 10°) and higher temperatures (15°, 20°). It's obviously that they form a complete set of orthogonal contrasts.

(d) The SAS output for testing the above contrasts is shown below.

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
C1	1	0.40000000	0.40000000	0.06	0.8049
C2	1	0.10000000	0.10000000	0.02	0.9016
C3	1	92.45000000	92.45000000	14.73	0.0024

Contrasts $\mathbf{C}_1$ and $\mathbf{C}_2$ are not significant, while contrast $\mathbf{C}_3$ is significant. This confirms the belief of the company that there is a jump in the strength at some temperature (12.5°) between 10° and 15°. ∎

Problem 3

(a) To do Tukey's Additivity test, I first fit an additive model and obtain the predicted values $\hat{y}_{ij}$, then add $q_{ij} = \hat{y}_{ij}^2$ to the predictor set and fit another model. Then Tukey's test is equivalent to the significance test of q. Here is the SAS output

Source	DF	Type III SS	Mean Square	F Value	Pr > F
q	1	8.19424514	8.19424514	3.85	0.1070

Since q is not significant (p-value$= 0.1070$), I conclude that there're no interaction effects between detergent and stain, and the additive model is valid. ∎