Homework 6 Solution

Problem 1

(a) Again, let's check the ANOVA table.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	489740.1875	163246.7292	12.73	0.0005
Error	12	153908.2500	12825.6875		
Corrected Total	15	643648.4375			

Since the p-value is very small ($= 0.0005$), I conclude that mixing techniques affect the strength of the cement.

(b) The SAS outputs for the four methods are listed below.

t Tests (LSD) for tensile

NOTE: This test controls the Type I comparisonwise error rate, not the experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	12
Error Mean Square	12825.69
Critical Value of t	2.17881
Least Significant Difference	174.48

Means with the same letter are not significantly different.

t Grouping	Mean	N	mixing
A	3156.25	4	2
B	2971.00	4	1
B			
B	2933.75	4	3
C	2666.25	4	4

Tukey's Studentized Range (HSD) Test for tensile

NOTE: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	12
Error Mean Square	12825.69
Critical Value of Studentized Range	4.19852
Minimum Significant Difference	237.74

Means with the same letter are not significantly different.

Tukey Grouping	Mean	N	mixing
A	3156.25	4	2
A			
A	2971.00	4	1
A			
A	2933.75	4	3
B	2666.25	4	4

Bonferroni (Dunn) t Tests for tensile

NOTE: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	12
Error Mean Square	12825.69
Critical Value of t	3.15268
Minimum Significant Difference	252.47

Means with the same letter are not significantly different.

Bon Grouping	Mean	N	mixing
A	3156.25	4	2
A			
A	2971.00	4	1
A			
A	2933.75	4	3
B	2666.25	4	4

Scheffe's Test for tensile

NOTE: This test controls the Type I experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	12
Error Mean Square	12825.69
Critical Value of F	3.49029
Minimum Significant Difference	259.13

Means with the same letter are not significantly different.

Scheffe Grouping	Mean	N	mixing
A	3156.25	4	2
A			
A	2971.00	4	1
A			
A	2933.75	4	3
B	2666.25	4	4

The critical differences for the four methods are

Method	LSD	Tukey	Bonferroni	Scheffe
CD	174.48	237.74	252.47	259.13

(c) The grouping results for each method are listed in the following table, where the treatments within one group are considered as not having significantly different effects and the treatments in different groups are considered as having significantly different effects.

Method	Grouping Results
LSD	$\{2\}, \{1, 3\}, \{4\}$
Tukey	$\{2, 1, 3\}, \{4\}$
Bonferroni	$\{2, 1, 3\}, \{4\}$
Scheffe	$\{2, 1, 3\}, \{4\}$

The smaller the critical difference is, the more power the method is. The bigger the critical difference is, the more conservative the method is. Hence, among the four methods, LSD is the most powerful one and Scheffe's method is the most conservative one. However, LSD does not control the over error rate, the other three control the over error rate. Apparently, Tukey's method should be preferred. ∎

Problem 2

(a) From the result of Problem 1, Assignment 3, $MS_E = 0.1325$ with $DF = 60$. This is a balanced ANOVA design with $a = 5$ and $n = 13$. We have $m = 4$ mean effect differences, so the critical distance for the Bonferroni method is

$$t_{\alpha/2m}(N - a)\sqrt{2MS_E/n} = t_{0.05/8}(65 - 5)\sqrt{2 * 0.1325/13} = 0.368.$$

Also, the control mean is 11.54 and the treatment means are $11.00, 11.42, 11.44, 11.28$ respectively. Hence the testing results for the Bonferroni method are

Pesticide	1	2	3	4
|Mean Diff|	0.54	0.12	0.10	0.26
Significance	Yes	No	No	No

(b) From Table IX in Montgomery, $d_{0.05}(4, 60) = 2.51$. So the critical distance for the Dunnet method is

$$d_\alpha(a - 1, N - a)\sqrt{2MS_E/n} = d_{0.05}(4, 60)\sqrt{2 * 0.1325/13} = 0.358.$$

Hence the testing results for the Dunnet method is

Pesticide	1	2	3	4
|Mean Diff|	0.54	0.12	0.10	0.26
Significance	Yes	No	No	No

(c) The significance test results for the treatment-control differences are the same in (a) and (b). In our test, the Bonferroni method is slightly more conservative than the Dunnet method, whereas the latter has more power. ∎