

Assignment 2 Answer Keys

1. Because the two samples are paired, the paired t -test should be used. The differences are

$$\begin{array}{ccccccccc} d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 & d_8 & d_9 \\ 0.125 & 0.159 & 0.259 & 0.277 & 0.135 & 0.224 & 0.328 & 0.451 & 0.507 \end{array}$$

a).

$$H_0 : \mu_1 = \mu_2 \text{ vs } H_1 : \mu_1 \neq \mu_2$$

or equivalently

$$H_0 : \mu_d = 0 \text{ vs } H_1 : \mu_d \neq 0$$

Based on d_1, \dots, d_9 , $\bar{d} = 0.274$, $s_d = 0.135$. The observed t -statistic is

$$t_0 = \frac{\bar{d}}{s_d/\sqrt{n}} = 6.08.$$

Because $t_{0.025,8} = 2.306$ and $t_0 > t_{0.025,8}$, we reject H_0 .

b)

$$P\text{-value} = P(t \geq t_0 \text{ or } t \leq -t_0 \mid H_0) = 0.0003$$

c) The 95% confidence interval for μ_d is

$$\begin{aligned} & (\bar{d} - t_{\alpha/2, n-1} s_d / \sqrt{n}, \bar{d} + t_{\alpha/2, n-1} s_d / \sqrt{n}) \\ &= (0.274 - 2.306 \times 0.135 / \sqrt{9}, 0.274 + 2.306 \times 0.135 / \sqrt{9}) \\ &= (0.170, 0.378) \end{aligned}$$

d) e): QQplots should be generated. There does not appear to be any significant departure from normality.

f) Blocking was used. Blocking can efficiently eliminate the variation due to girders and increase the power to detect the difference between the two measurements. On the other hand, blocking will result in reduced degree of freedom, and reduce the power. The success

of blocking depends on the trade-off between variation elimination and DF loss.

2. Because the population variance is known to be $\sigma^2 = 100$, only z -test is needed. The test statistic is

$$Z = \frac{\bar{X} - 200}{\sigma/\sqrt{n}}.$$

The decision rule is that

$$\text{Reject } H_0, \text{ if } Z > z_{0.05} = 1.645$$

or equivalently

$$\text{Reject } H_0, \text{ if } \bar{X} > 200 + 1.645 \frac{10}{\sqrt{n}}.$$

The probability of type II error is

$$\beta = P(\text{accept } H_0 \mid H_1) = P(\bar{X} \leq 200 + 1.645 \frac{10}{\sqrt{n}} \mid H_1)$$

In particular, the manufacturer wants to control β for $\mu \geq 210$. She requires that

$$\beta = P(\bar{X} \leq 200 + 1.645 \frac{10}{\sqrt{n}} \mid \mu = 210) \leq 5\%.$$

Under $\mu = 210$, \bar{X} follows $N(210, \frac{10}{\sqrt{n}})$. So,

$$\beta = P\left(\frac{\bar{X} - 210}{10/\sqrt{n}} \leq -\sqrt{n} + 1.645\right) \leq 0.05.$$

Hence,

$$-\sqrt{n} + 1.645 \leq -1.645,$$

and

$$n \geq (1.645 * 2)^2 = 10.8.$$

She has to check at least 11 lots.