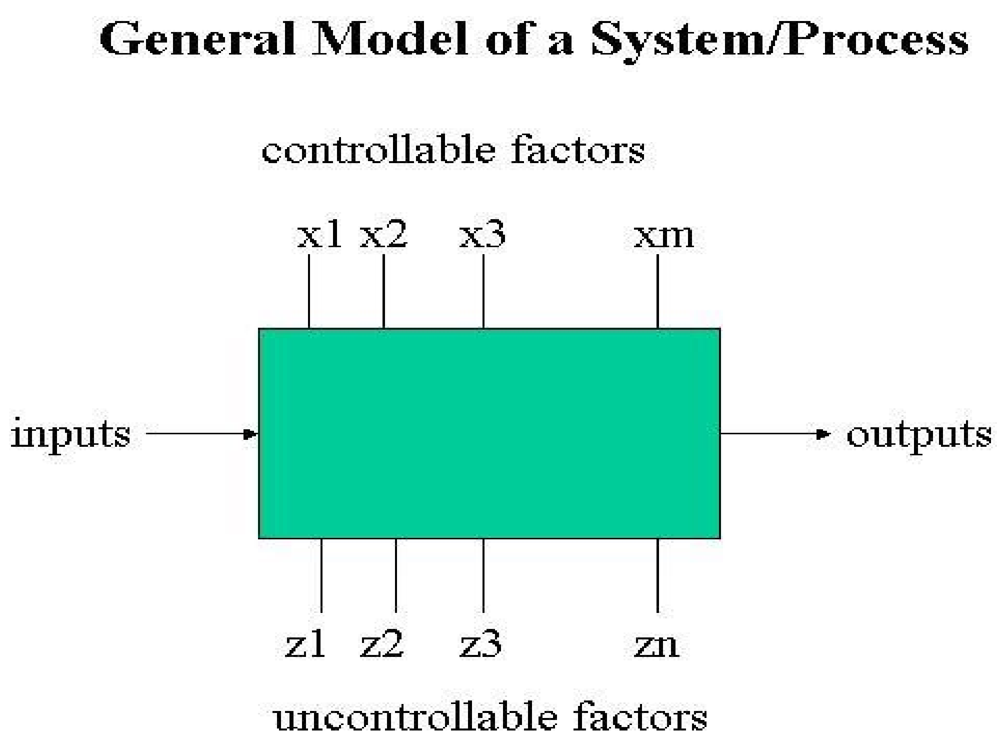
Lecture 1. Overview and Basic Principles

Montgomery: Chapter 1


General Model of a System/Process
controllable factors
x1 x2 x3 xm
inputs
outputs
z1 z2 z3 zn
uncontrollable factors

Statistical Methods for Design and Analysis of Experiments

1. Experimental error.
2. Confusion of correlation with causation.
3. Complexity of the effects in study.

Some Terminologies:

Experimental factor (or variable)

Factor level

Treatment (setting or level combination)

Unit

Experimental run (trial)

Experimental error

Machine Tool Life Experiment

An engineer is interested in the effects of **cutting speed** (A), **tool geometry** (B) and **cutting angle** (C) on the lifespan (in hours) of a machine tool. Two levels of each factor are chosen and three replicates of a 2^3 factorial design are run. The results follow.

Factor			Replicate		
A	B	C	I	II	III
$-$	$-$	$-$	22	31	25
$+$	$-$	$-$	32	43	29
$-$	$+$	$-$	35	34	50
$+$	$+$	$-$	55	47	46
$-$	$-$	$+$	44	45	38
$+$	$-$	$+$	40	37	36
$-$	$+$	$+$	60	50	54
$+$	$+$	$+$	39	41	47

A Brief History of Experimental Design

1. Agricultural Era:

   -Treatment Comparisons and ANOVA

   -R.A. Fisher, Rothamsted Agricultural Experimental Station (1930, England)

   -Introduced statistical experimental design and data analysis

   -Summarized the fundamental principles: replication, randomization, and blocking.

   -An influential book, *The Design of Experiments*

Combinatorial Design Theory: R. C. Bose

2. Industrial Era:

   -Process modeling and optimization

   -Box and coworkers in chemical industries and other processing industries

   -Empirical modeling, response surface methodologies, central composite design

Optimal designs: J. Kieffer

3. Quality Era:

   -Quality improvement and variation reduction

   -Taguchi and robust parameter design

   -Statistical design and analysis toward robustness.

4. Current State of Experimental Design:

-Popular outside statistics, and an indispensable tool in many scientific/engineering endeavors

-New challenges:

–Large and complex experiments, e.g., screening design in pharmaceutical industry, experimental design in biotechnology

–Computer experiments: efficient ways to model complex systems based on computer simulation.

–. . .

Classify Experiments Based on Experimental Objectives

1. Variable screening.
2. Treatment comparison.
3. Response surface exploration.
4. System optimization
5. System robustness.

A Systematic Approach to Experimentation

1. State objectives.

2. Choose responses.

-What to measure? How to measure? How good is the measurement system?

-Measurement systems capability study.

3. Choose factors and levels

-Flow chart and cause-and-effect diagram.

-Factor experimental range is crucial for success

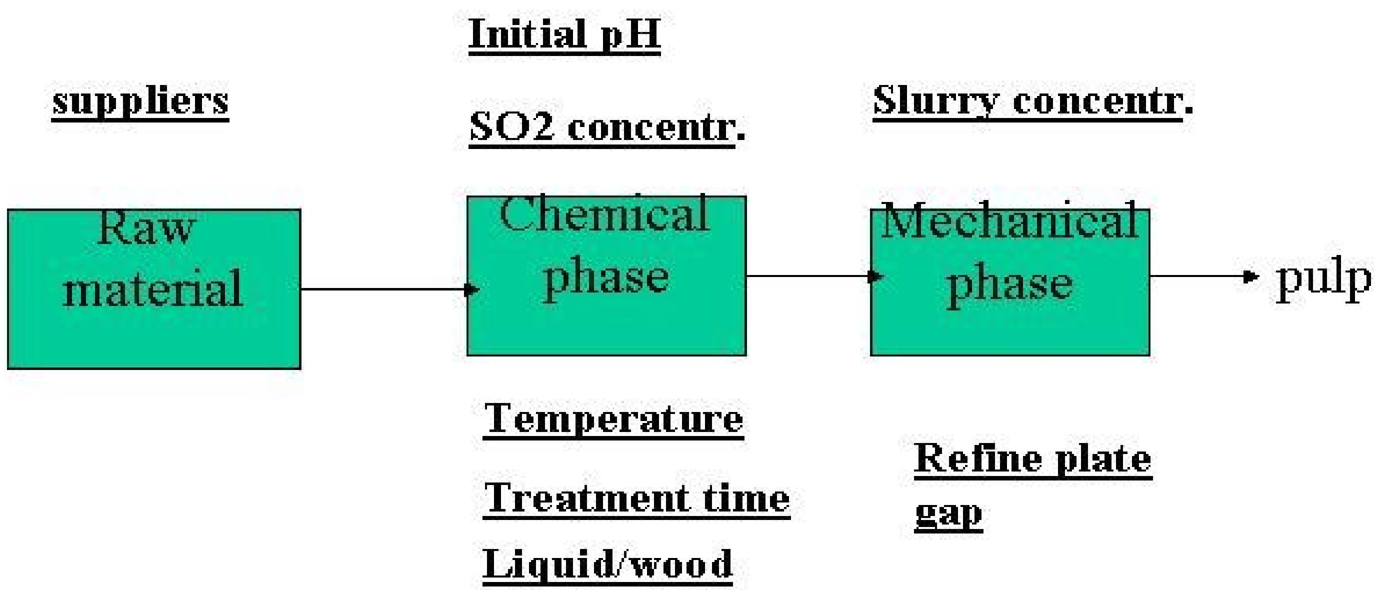
4. Choose experimental plan.
5. Conduct the experiment.
6. Analyze the data

7. Conclusion and recommendation.

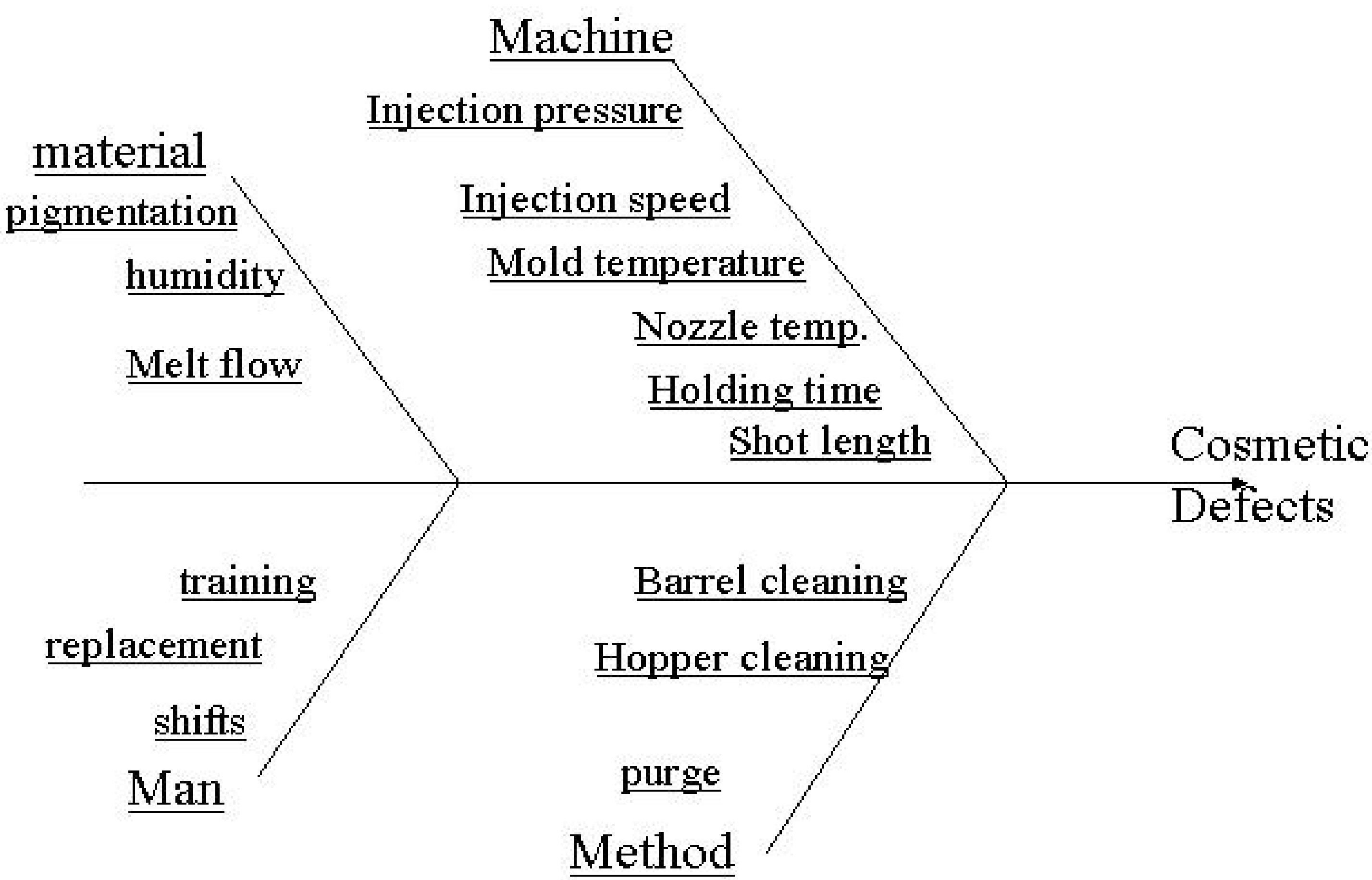
-iterative procedure

-confirmation experiments/follow-up experiments

Flow Chat, Pulp Manufactoring Process



Cause-and-Effect Diagram, Injection Molding Experiment



Fundamental Principles: Replication, Randomization, and Blocking

1. Replication

1. Each treatment is applied to a number of units representative of the population (of units)

2. Enable the estimation of experimental error.

3. Increase the power to detect important effects

4. Replication vs Repetition (or repeated measurements)

2. Randomization

1. Allocation (of treatments to units), run order and measurement order need to be randomized.

2. Protect against latent variables and subjective biases.

3. Ensure the validity of experimental error estimation.

4. Ensure the validity of statistical inferences.

5. Complete randomization makes possible to derive and perform

Randomization Distribution and Randomization Test

A Randomized Experiment: Modified Fertilizer Mixtures for Tomato Plants

An experiment was conducted by an amateur gardener whose object was to discover whether a change in the fertilizer mixture applied to his tomato plants would result in an improved yield. He had 11 plants set out in a single row; 5 were given the standard fertilizer mixture A, and the remaining 6 were fed a supposedly improved mixture B. The A's and B's were randomly applied to the positions in the row to give the design shown in next slide. The gardener arrived at this random arrangement by taking 11 playing cards, 5 red corresponding to fertilizer A and 6 black corresponding to fertilizer B. The cards were thoroughly shuffled and dealt to give the sequence shown in the design. The first card was red, the second was red, the third was black, and so forth.

Fertilizer Mixtures Experiment: Design and Results

Pos	1	2	3	4	5	6	7	8	9	10	11
Trt	A	A	B	B	A	B	B	B	A	A	B
Yds	29.9	11.4	26.6	23.7	25.3	28.5	14.2	17.9	16.5	21.1	24.3

A	B
29.9	26.6
11.4	23.7
25.3	28.5
16.5	14.2
21.1	17.9
	24.3

$n_A = 5 \qquad n_B = 6$

$\Sigma y_A = 104.2 \qquad \Sigma y_B = 135.2$

$\bar{y}_A = 20.84 \qquad \bar{y}_B = 22.53$

Mean difference (modified minus standard)= $\bar{y_B} - \bar{y_A} = 1.69$

Testing Hypotheses

H_0: the modified fertilizer does not improve the (mean) yield.

H_a: the modified fertilizer improves the (mean) yield.

Under the null hypothesis, A and B are mere labels and should not affect the yield. For example, the first plant would yield 29.9 pounds of tomatoes no matter it had been labeled as A or B (or fed A or B).

There are $\frac{11!}{5!6!} = 462$ ways of allocating 5 A's and 6 B's to the 11 plants, any one of which could equally be chosen. The used design is just one of 462 equally likely possibilities. (why?)

For example:

Pos	1	2	3	4	5	6	7	8	9	10	11
Yds	29.9	11.4	26.6	23.7	25.3	28.5	14.2	17.9	16.5	21.1	24.3
LL1	A	A	A	A	A	B	B	B	B	B	B
LL2	A	A	A	A	B	A	B	B	B	B	B
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

LL1, LL2, etc are equally likely.

LL1: mean difference between B and A is -2.96

LL2: mean difference between B and A is -4.14

⋮

Under the null hypothesis, these differences are equally likely.

An Aside: Hypothesis Testing: Criminal Trial Analogy

1 Two Hypotheses:

H_0 : Defendant not guilty (innocent assumption) H_a : Defendant guilty

Note: H_0 represents the status quo. H_a is the conclusion that the persecution (researcher) tries to make.

2 . Collecting evidence:

- In trail, finger prints, blood spots, hair samples, carpet fibers, shoe prints, ransom notes, etc.
- In testing, survey, experiment, **data** .

3 . Fundamental Assumption:

- In trial, defendant is innocent until proven guilty, i.e. H_0 is assumed to be true.
- In testing, similarly, we always assume H_0 is true.

4 . Summarizing Evidence:

- In trial: Cross examination, argument, jury deliberation.
- In testing: test statistic, its sampling distribution (under H_0), and observed test statistic.

5 . Decision Rule:

- In trial: Reject H_0, if beyond a reasonable doubt (under the innocent assumption).
- In testing: Reject H_0, if the observed test statistic is extreme enough: more extreme than a critical value or its P-value is less than a threshold.

6 . An Important Point

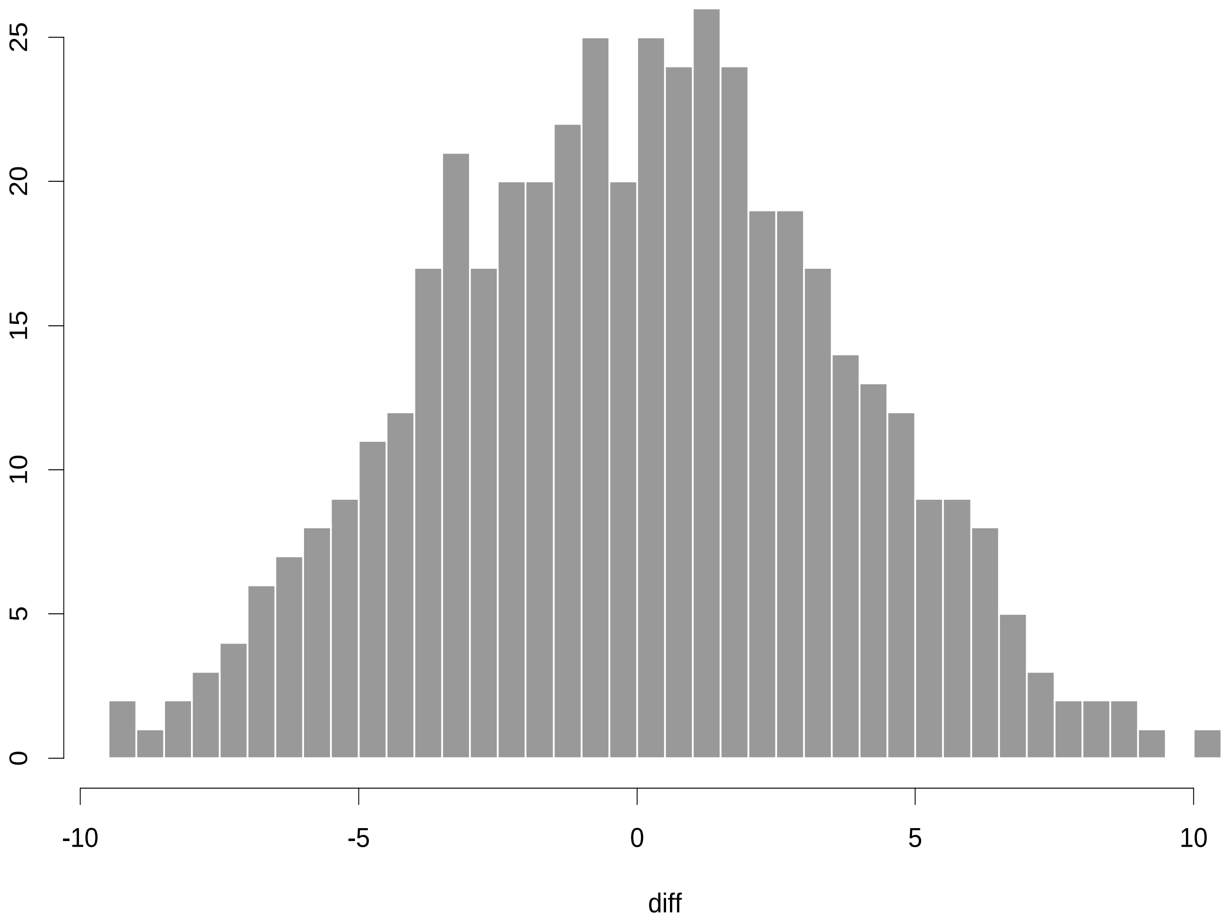
Neither decision entails proving H_0 or H_a. We merely state there is enough evidence to behave one way or the other. This is true in both trial and testing. No matter what decision we make, there is always a chance we made an error.

Significance of Observed Difference

A summary of possible allocations and their corresponding mean differences:

No	possible designs	$\bar{y}_A$	$\bar{y}_B$	mean difference
1	AAAAABBBBBB	23.38	20.42	-2.96
2	AAAABABBBBB	24.02	19.88	-4.14
⋮	⋮	⋮	⋮	⋮
·	AABBABBBAAB	20.84	22.53	1.69
⋮	⋮	⋮	⋮	⋮
462	BBBBBBAAAAA	18.80	24.23	5.43

Randomization Distribution (Histogram) of the Mean Differences



$$H_0 : \mu_A = \mu_B \text{ vs. } H_a : \mu_B > \mu_A (\alpha = 5\%)$$

1. Randomization Test:

   Observed Diff $= 1.69$

   P-value $= Pr(\text{Diff} \geq 1.69 \mid \text{randomization}) = \frac{155}{462} = .335$

   Because P-value $\geq \alpha$, accept H_0.

2. Two sample t-test (refer to Page 36 of Montgomery)

   $s_A^2 = 52.50, s_B^2 = 29.51$

   $s_{pool}^2 = \frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2} = 39.73$

   $t_0 = \frac{\bar{y}_B - \bar{y}_A}{s_{pool}\sqrt{1/n_A + 1/n_B}} = .44$

   P-value $= Pr(t > t_0 \mid t(n_A + n_B - 2)) = Pr(t > .44 \mid t(9)) = .34$

   Because P-value $\geq \alpha$, accept H_0.

Blocking

1. A **block** refers to a group of homogeneous units/runs.

2. Within-block variation and between-block variation

3. Trade off between variation and the degrees of freedom

Block what you can and randomize what you cannot

Randomization and Blocking, Typing Efficiency Experiment

Compare the typing efficiency of two keyboards denoted by A and B. One typist uses the keyboards on six different manuscripts, denoted by 1-6.

Design 1:

$$1. A - B, 2. A - B, 3. A - B, 4. A - B, 5. A - B, 6. A - B.$$

Design 2:

$$1. A - B, 2. B - A, 3. A - B, 4. B - A, 5. A - B, 6. A - B.$$

Design 3: Balanced Randomization (3 $A - B$s and 3 $B - A$s)