

Stat 514 Design of Experiments Midterm Exam Spring 2025

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Name:

Section (Circle One): 11:30-12:30

12:30-1:30

Solution

To receive full credit:

Show all work that lead to the final answer on the attached pages

Please write your answer on the back of a page if need more space

Total 30 points

Use $\alpha = 0.05$ for all the exam problems.

Problem 1 (15): In an experiment to investigate the effect of color of paper (1=green, 2=yellow, 3=red) on response rates for questionnaires distributed by the "windshield method" in supermarket parking lots, 18 representative supermarket parking lots were chosen in a metropolitan area and each color was assigned at random to five of the lots. The response rates (in percent) are given below.

Color	Response Rate					
1=green	49	47	52	48	53	45
2=yellow	55	50	46	52	50	49
3=red	47	39	41	45	44	37

1. (5) Fill in the missing values in the following ANOVA table.

	DF	Sum-of-Squares	Mean-Square	F-Value	Pr > F
Model	2 ??	238.11 ??	119.055 ??	?? 9.73	0.0020
Error	15 ??	183.50	12.233 ??		
Total	17 ??	421.61			

Normality; equal variance;
independence

2. (1) What are the three assumptions for the ANOVA model??
3. (1) What is the estimate of the variance σ^2 ? $\hat{\sigma}^2 = 12.233$
4. (2) For pairwise comparison using Tukey's method, we have the following SAS output. What is the half interval size (Minimum Significant Difference)?

Alpha	0.05
Error Degrees of Freedom	?? 15
Error Mean Square	?? 12.233
Critical Value of Studentized Range	3.67338
Minimum Significant Difference	?? 5.2452 = 3.67338 / $\sqrt{2} \times \sqrt{12.233 \times \frac{1}{6} + \frac{1}{6}}$

5. (3) Based on the Tukey's method above, compute the 3 confidence intervals for $\mu_1 - \mu_2$, $\mu_1 - \mu_3$, $\mu_2 - \mu_3$? (Sample means are 49.33 (green), 50.33 (yellow), 42.17 (red)).
6. (3) We test these two orthogonal contrasts. And got the SAS output. Compute the missing values.

```
contrast 'c1' color 1 -1 0;
contrast 'c2' color 1 1 -2;
```

Contrast	DF	Contrast-SS	Mean-Square	F-Value	Pr > F
c1	?? 1	3.00	3 ??	?? 0.25	0.6267
c2	?? 1	??	??	??	0.0005

235.11 235.11 19.22

⑤ $\mu_1 - \mu_2: (49.33 - 50.33) \pm 5.2452 = (-6.2452, 4.2452)$

$\mu_1 - \mu_3: 7.16 \pm 5.2452 = (1.9148, 12.4052)$

$\mu_2 - \mu_3: 8.16 \pm 5.2452 = (2.9148, 13.4052)$

Problem 2 (15): A city highway department studied the wear characteristics of four different paints (paints A, B, C, D) at 4 locations. The 4 locations were randomly selected, thus reflecting variations in traffic conditions in the city. At each location, a random ordering of the paints to the chosen road surface was employed. After a period of exposure to weather and traffic, a measure of wear was obtained for each paint. The data on wear and various means are given below.

paint	location			
	I	II	III	IV
A	27	13	32	19
B	33	19	37	23
C	20	23	31	18
D	25	19	32	21

1. (1) What type of design was used in this experiment?

Randomized Complete Block Design

2. (1) What is the block factor in this experiment?

location.

3. (1) Write down the model for this design.

resp = loc paint; $y_{ijk} = \mu + \tau_i + \beta_j + \epsilon_{ijk}$

4. (1) What are the assumptions for your model?

Normality; equal variance; independence

5. (5) Compute the missing values in the ANOVA table

Source	DF	Sum-of-Squares	Mean-Square	F-Value	Pr > F
Model	6	?? 588	?? 98	??	0.0027
Error	9	104.00	?? 11.555	8.48	
Total	15	692.00			

6. (3) Compute the missing values in the table for Type III SS.

Source	DF	Type-III-SS	Mean-Square	F-Value	Pr > F
paint	3	70.50	23.5	2.03	0.1797
loc	3	517.5	172.5	14.93	0.0008

7. (2) For pairwise comparison using Bonferroni method, we have the following SAS output.

What is the half interval size (Minimum Significant Difference)?

Alpha	0.05
Error Degrees of Freedom	?? 9
Error Mean Square	?? 11.555
Critical Value of t	3.36420
Minimum Significant Difference	?? 8.0865 = 3.3642 * $\sqrt{11.555} \times \sqrt{\frac{1}{4} + \frac{1}{4}}$

8. (1) For a 3x3 Latin Square Design, fill up the missing design column below

C	B	??	A
B	A	??	C
A	C	??	B