STAT 417—Practice Midterm

Name:________________________ **Section:**________________________

1. (10 Points) Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution $N(0, \theta)$ with mean 0 and variance θ, where $0 < \theta < \infty$. The $N(0, \theta)$ pdf is given by

$$f(x; \theta) = \frac{1}{\sqrt{2\pi\theta}} e^{-\frac{x^2}{2\theta}}, \quad -\infty < x < \infty.$$

(a). Find the MLE $\hat{\theta}$ of θ.

(b). Is the MLE $\hat{\theta}$ an unbiased estimator of θ? Why?

$\Big[$If $Z \sim N(0, 1)$, then $Z^2 \sim \chi_1^2$. The mean and variance of a χ_1^2 random variable are 1 and 2, respectively.$\Big]$

2. (10 Points) $X \sim N(\mu, 1)$. We want to estimate $\theta = E(X^2)$ on the basis of a random sample $X_1, X_2, \ldots, X_n$. Determine the MLE of θ and show that it is biased. Can you construct an unbiased estimator of θ?

3. (10 Points) In a study of anemia in cattle, researchers measured concentration of selenium in blood of 36 cows that had been given a dietary supplement of selenium. The cows were of the same breed (Santa Gertrudis) and had borne their first calf during the year. The mean selenium concentration was 6.21 g/mLi and the standard deviation was 1.84 g/mLi.

a) Construct a 95% confidence interval for the population mean.

b) Estimate how many cows should be used in the study for the standard error of the sample mean to be smaller than 0.2 g/mLi?

4. (10 Points) Let $X_1, X_2, \ldots, X_n$ be a random sample from a Gamma population with the **KNOWN** parameter α, where the pdf of Gamma(α, β) is

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, \quad x > 0, \quad \beta > 0.$$

The mean and the variance of a Gamma random variable are $E(X_1) = \alpha\beta$ and $Var(X_1) = \alpha\beta^2$, respectively.

(a). Find the maximum likelihood estimator (MLE) for β.

(b). Is the MLE in (a) an unbiased estimator of β? Why?

(c). Find the MLE for $\tau = (2\beta - 1)^2$.

5. (10 Points) A polling firm conducts a poll to determine what proportion θ of voters in a given population will vote in an upcoming election. A random sample of $n = 250$ was taken from the population and 155 answered yes. Assess the hypothesis $H_0 : \theta = 0.65$ and report the P-value. Construct an approximate 0.90-confidence interval for θ.