STAT 417—Practice Final

Name:_____ **Section:**_____

1. (30 Points) A botanist was interested in the effect the amount of water given to a plant has on the length of its root system. She selected a random sample of 32 seeds of a particular variety of plant. She grew plants from these seeds in a greenhouse. All plants were kept under identical conditions, except that the plants were given different amounts of water, and the total amount of water, x, given to each plant was recorded. The average amount of water given to the plants was 48.6 L, and the standard deviation of the water given was 9.58 L. At the end of a specific growing time, the plants were carefully uprooted. For each plant, the distance, y, from soil level to the farthest root tip was measured. The average root distance in the 32 plants was 20.0 cm, with standard deviation 3.31 cm. For each plant, the product of root distance times total water was calculated, and the sum of these values over all plants was $\sum_{i=1}^{32} x_i y_i = 30,382.01$ L cm.

(a) Find the line of the best fit that describes the effect of total water on root distance.

(b) Based on the model in (a), what root distance would you predict for a plant given 46.3 L of water?

(c) What percentage of the variation in root distance is explained by regression the total water?

2. (15 Points) Using the data shown in the following table to test at the 0.01 level of significance whether a person's ability in mathematics is independent of his or her interest in statistics.

		Ability in Math		
		Low	Average	High
	Low	63	42	15
Interest in Stat	Average	58	61	31
	High	14	47	29

3. (15 Points) Suppose that the following sample is assume to be from $N(\theta, 2)$ distribution: $-0.4, 1.9, -0.3, -0.2, 0.0, 0.0, -0.1, -0.1, 2, 4$. Check this model for the normality assumption.

(a) Assume $\theta = 0$.

(b) Assume θ is unknown.

4. (15 Points) Suppose that a random sample of size n is from Uniform$[0, \theta]$, where $\theta > 0$ is unknown. We have a Gamma(α_0, β_0) prior for θ.

(a) What is the MLE of θ?

(b) What is the posterior model of θ?

5. (15 Points) If X is a binomial random variable $Bin(n; \theta)$ and the prior distribution of θ is a beta distribution with parameters α and β.

(a) What is the posterior distribution of θ given $X = x$?

(b) How to construct a 95% credible interval for θ?

(c) What is the posterior mode? Is this the same as the MLE of θ?

6. (10 Points) Let $X_1, \ldots, X_n$ be a random sample from $N(0, \sigma^2)$, where $\sigma^2 > 0$ is unknown. The prior distribution $1/\sigma^2 \sim Gamma(\alpha_0, \beta_0)$ is appropriate. Assess the hypothesis $H_0 : \sigma^2 \leq 1$ by calculating the posterior probability.