STAT 417. Lecture 1. Review of probability.

Definition: A is an event. The probability of A is

$P(A)$ = the proportion of times that event A occurs in a sequence of experiments.

Three Axioms: ① $0 \le P(A) \le 1$. for any event A.

② $P(S) = 1$. S is the sample space.

③. $A \cap B = \phi$. then. $P(A \cup B) = P(A) + P(B)$.

Combination and permutation:

permutation: A set has n elements. Select k elements from the set without replacement. and arrange the k elements in a line.

Each arrangement is called a k-permutation.

the total number of k-permutations is $P_{n,k} = \frac{n!}{(n-k)!}$

Combination: If the order does not matter. any selection is called a combination. Total number of combinations is $\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$

Eg 1. Choose two different dates from the year 2014. What is the probability that both dates are in January.?

sol: $\frac{\binom{31}{2}}{\binom{365}{2}} = 0.0069998.$

Eg2. Draw 2 cards from 52 cards without replacement.

what is the probability that the first is a King and the second is a Queen?

$$\frac{\binom{4}{1}\binom{4}{1}}{P_{52,2}} = \frac{4\times 4}{52\times 51} = 0.00603$$

~~Eg3. Rolling a die, twice, what is the probability that the~~

Conditional probability: $P(A|B) = \frac{P(A\cap B)}{P(B)}$.

Independence: A is independent of B if $P(A|B) = P(A)$, or equivalently,

$$P(A\cap B) = P(A)\,P(B).$$

Eg3. Rolling a die twice. What is the probability that, given the first roll is a six, the second roll is also a six?

Sol: A = the 1st roll is six, B = the 2nd roll is a six. A is independent of B.

$P(B|A) =$ $P(B) = \frac{1}{6}$.

Note: $P(A\cap B) = 0$ does not imply A and B are independent.

Some useful facts: $P(A\cup B) = P(A) + P(B) - P(A\cap B)$.

if A and B are independent, then $P(A\cup B) = P(A) + P(B) - P(A)P(B)$.

if $P(A\cap B) = 0$, then $P(A\cup B) = P(A) + P(B)$.

Eg.4. Draw two cards from 52 cards with replacement.

Then p(first is K or second is Q).

$= p(\text{first is K}) + p(\text{second is Q}) - p(\text{first is K})\, p(\text{second is Q})$

$= \frac{4}{52} + \frac{4}{52} - \left(\frac{4}{52}\right)\left(\frac{4}{52}\right) = 0.14792.$

Random variable: Discrete. Binomial, poisson.

Continuous. Normal, Exponential, Gamma.

Eg.5. Rolling a die three times. Independently.

$p(1, 1, 2) = p(1)\, p(1)\, p(2) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{216},$

p(the tree outcomes are 1, 1, 2 in any order)

$= 3! \left(\frac{1}{216}\right) = \frac{1}{36}.$