Testing Hypothesis and P-values

- Null hypothesis $H_0: \psi(\theta) = \psi_0$

If we decide that the data are suprising under H_0 then this is evidence against H_0.

$\longrightarrow$ P-value: is a probability which is a measure of surprise.

Small p-value $\longrightarrow$ a suprising event has occurred ~~wif~~ H_0 is true.

$\longrightarrow$ reject H_0.

P-value is not the probability that H_0 is true.

- Z-tests.

$X_1 \cdots X_n \overset{iid}{\sim} N(\mu, \sigma_0^2)$. σ_0^2 is known

$H_0: \mu = \mu_0$. $H_1: \mu \neq \mu_0$.

Under H_0:

$$Z = \frac{\bar{X} - \mu_0}{\sigma_0/\sqrt{n}} \sim N(0,1) \quad \text{or} \quad \bar{X} \sim N\left(\mu_0, \frac{\sigma_0^2}{n}\right)$$

$$\text{P-value} = P(|Z| \geq |z|) = 2\left[1 - \Phi\left(\left|\frac{\bar{x} - \mu_0}{\sigma_0/\sqrt{n}}\right|\right)\right]$$

ex. We generate.

$n = 10$. $N(26, 4)$.

29.0651	27.3980	23.4346	26.3665	23.4994
28.6592	25.5546	29.4477	28.0979	25.2850

$H_0: \mu = 25$.

p value: $2\left[1 - \Phi\left(\left|\frac{\bar{X} - \mu_0}{\sigma_0/\sqrt{n}}\right|\right)\right] = 2(1 - \Phi(2.6576)) = 0.0078$.

reject H_0.

ex. $X_1 \cdots X_n \overset{iid}{\sim} \text{Bernoulli}(\theta)$.

$H_0: \theta = \theta_0$.

$$Z = \frac{\sqrt{n}(\bar{X} - \theta_0)}{\sqrt{\theta_0(1-\theta_0)}} \sim N(0,1) \quad \text{under } H_0.$$

p value: $2\left[1 - \Phi\left(\left|\frac{\sqrt{n}(\bar{X} - \theta_0)}{\sqrt{\theta_0(1-\theta_0)}}\right|\right)\right]$.

obs 54 among 100 tosses. $\hat{\theta} = 0.54$.

$H_0: \theta = \frac{1}{2}$

$$2\left(1 - \Phi\left(\left|\frac{\sqrt{100}(0.54 - 0.5)}{\sqrt{0.5(1-0.5)}}\right|\right)\right) = 2(1 - \Phi(0.8)) =$$

$$2(1 - 0.7881) = 0.4238$$

accept H_0.

- T-test.

$X_1 \cdots X_n \overset{iid}{\sim} N(\mu, \sigma^2)$.

$H_0: \mu = \mu_0$. $\quad H_1: \mu \neq \mu_0$.

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1} \quad \text{under } H_0 .$$

p-value: $P(|T| \geq |t|)$ $\quad$ T-table

ex.

$\bar{X} = 26.6808$

$S = \sqrt{4.8620} = 2.2050$

$H_0: \mu = 25$.

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{26.6808 - 25}{2.2050/\sqrt{10}} = 2.4105$$

$t_{0.0975, 9} = 2.2622$. $\quad$ p-value: 0.039 $\quad$ reject H_0