STAT417. Lecture Note 6.

E.g.2. Toss a coin $n=10$ times. $\theta = P(H)$. So, the number of heads X follows Binomial$(10, \theta)$. Suppose $s=4$ heads were observed.

Find $L(\theta | s)$.

Sol: $L(\theta|s) = f_\theta(s) = \binom{n}{s} \theta^s (1-\theta)^{n-s} = \binom{10}{4} \theta^4 (1-\theta)^6$

~~[illegible]~~. When data s has ~~more~~ more than one sample.

E.g.3. Suppose $s=(X_1, X_2)$ is an i.i.d sample from $N(\mu, \sigma^2)$.

Find ~~$L(\mu, \sigma^2 | s)$~~. $L(\theta|s)$, where $\theta = (\mu, \sigma^2)$.

Sol: ~~$L(\mu, \sigma^2 | s) = f_{\mu,\sigma^2}(X_1, X_2) =$~~

$$L(\theta|s) = \cancel{f_\theta(s)} f_\theta(X_1, X_2) = f_\theta(X_1) f_\theta(X_2)$$

$$= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(X_1-\mu)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(X_2-\mu)^2}{2\sigma^2}}.$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^2 e^{-\frac{(X_1-\mu)^2+(X_2-\mu)^2}{2\sigma^2}}.$$

E.g.4. Suppose $s = (X_1, X_2, \cdots, X_n)$ is an i.i.d sample from $N(\mu, \sigma^2)$.

Find $L(\theta|s)$, where $\theta = (\mu, \sigma^2)$

sol:

$$L(\theta \mid S) = f_\theta(x_1, x_2, \dots, x_n) = f_\theta(x_1) f_\theta(x_2) \cdots f_\theta(x_n)$$

$$= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}} \cdots \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_n-\mu)^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n e^{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}}.$$

EX 1. Suppose X_1, X_2 are two samples (iid) from $Gamma(\alpha, \beta)$. recall $Gamma(\alpha, \beta)$ has pdf

$$f(x) = \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta^\alpha}, \quad x > 0.$$

Find the likelihood function $L(\theta \mid x_1, x_2)$, $\theta = (\alpha, \beta)$.

Eg 5. Suppose $X_1, X_2, \dots, X_n$ are iid samples from $unif(0, \theta)$.

Find $L(\theta \mid x_1, x_2, \dots, x_n)$.

sol: $L(\theta \mid x_1, x_2, \dots, x_n) = f_\theta(x_1, x_2, \dots, x_n)$

$$= f_\theta(x_1) f_\theta(x_2) \cdots f_\theta(x_n)$$

$$= \frac{1}{\theta} \mathbb{1}_{(0,\theta]}(x_1) \cdot \frac{1}{\theta} \mathbb{1}_{(0,\theta)}(x_2) \cdots \frac{1}{\theta} \mathbb{1}_{(0,\theta)}(x_n)$$

$$= \frac{1}{\theta^n} \prod_{i=1}^{n} \mathbb{1}_{(0,\theta)}(x_i).$$