

The Idea

We want to answer a scientific question about some population(s).

We use data (samples) to make our mind/decision.

Since we only have samples, not populations, we do not have all the information and therefore will sometimes make a wrong decision.

We may ask, e.g., the following questions about the population parameters.

For a population with a binomial distribution

Is the success probability equal to $1/2$, i.e. is the coin fair?

Is the success probability equal to p_0 , i.e. some specific value of interest?

For one normally-distributed population:

Is the population mean equal to 93?

Is the population mean equal to μ_0 , i.e. some specific value of interest?

For 2 populations with normal distributions:

Do the two populations have the same means (is the difference in population means equal to 0)?

Is the difference in population means equal to d , i.e. some specific value of interest?

We are going to “answer” these questions with one of two possible decisions.

Rather than “True” we will say, e.g.,

“The study does not provide evidence of a difference in the population means.”

Instead of “False” we will say, e.g.,

“The study provides evidence of a difference in the population means.”

Analogy: A person is on trial for murder.

- The prosecutor presents evidence against the accused.
 - The jury weights the evidence and announces a verdict.
 - The judicial system is not perfect, sometimes mistakes are made.
 - There are two possible situations: accused is innocent or guilty.
 - There are two possible decisions for the jury: declare not guilty or guilty.
 - People are presumed innocent until proven guilty beyond a reasonable doubt.
- Thus the default state is innocent.
- The opposite state represents the unusual or noteworthy case: guilty.
 - Jury has to make a decision based on the evidence.

Hypotheses

- We give a name to the “default state”, innocent: the null hypothesis.
- We give a name to the “interesting state”, guilty: the alternate hypothesis.
- H_0 is short for null hypothesis (H-zero or H-naught).
- H_A is short for alternate hypothesis (H-eh)
- H_0 is the default state innocent
- H_A is the interesting state guilty.
- The true state of nature is one of these two states.

Decisions

- The decision is based on the evidence given at the trial.
- It is always phrased in terms of H_0 .
- When the evidence is strong against the accused we declare they are guilty. We call this rejecting H_0 , i.e. we reject the idea that the person is innocent.
- When the evidence is weak or non-existent we declare not guilty. We call this not rejecting H_0 , i.e. we stick to the idea of innocence.
- Our basis for a decision is a summary of the evidence presented at the trial. Henceforth we will call this a “test statistic”.
- When the evidence is strong we call the test significant. A significant result causes us to reject H_0 . For clarity: “rejecting H_0 ” and “significant test” are synonyms.
- When the evidence is weak we say that it is not significant and we fail to reject H_0 .

Errors

- 4 things can happen:
 1. Declare an innocent person not guilty (good).
 2. Declare an innocent person guilty (type I error).
 3. Declare a guilty person not guilty (type II error).
 4. Declare a guilty person guilty (good).
- There are 2 good decision and two bad decisions.
- The name of the error is based on the true state is (innocent or guilty).
- Often these two types of errors have very different consequences or costs.
- Summarizing, there are two types of errors:
 1. Type I: rejecting H_0 when it is actually true,
 2. Type II: failing to reject H_0 when H_A is true.