1. Let $f(x)$ and $F(x)$ denote, respectively, the pdf and the distribution function of the random variable X. The conditional pdf of X given $X > x_0$, x_0 a fixed number, is defined by $f(x|X > x_0) = f(x)/[1 - F(x_0)]$, $x > x_0$, zero elsewhere. This kind of pdf finds application in a problem of time until death, given survival until time x_0.
(a) show that $f(x|X > x_0)$ is a pdf.
(b) Let $f(x) = e^{-x}$, $0 < x < \infty$, zero elsewhere. Compute $P(X > 2|X > 1)$.

2. If the correlation coefficient ρ of X and Y exits, show that $|\rho| \leq 1$. *Hint:* Consider the discriminant of the nonnegative quadratic function $h(v) = E\{[(X-\mu_1)+v(Y-\mu_2)]^2\}$, where v is real and is not a function of X nor of Y.

3. Let X and Y have the joint pdf $f(x, y) = e^{-2}/[x!(y - x)!]$, $y = 0, 1, 2, \ldots; x = 0, 1, \ldots, y$, zero elsewhere.
(a) Find the moment-generating function $M(t_1, t_2)$ of this joint distribution.
(b) Compute the means, the variances, and the correlation coefficient of X and Y.
(c) Determine the conditional mean $E(X|Y = y)$. *Hint:* Note that $\sum_{x=0}^{y}[\exp(t_1 x)]y!/[x!(y-x)!] = [1 + \exp(t_1)]^y$.

4. Let X_1 and X_2 denote a random sample of size 2 from a distribution with pdf $f(x) = 1$, $0 < x < 1$, zero elsewhere. Find the distribution function and the pdf of $Y = X_1/X_2$.

5. Let U and V be two independent random variables, each having a normal distribution with mean zero and variance one. Show that the moment generating function $E(e^{t(UV)})$ of the product UV is $(1 - t^2)^{-1/2}$, $-1 < t < 1$. *Hint:* Compare $E(e^{t(UV)})$ with the integral of a bivariate normal pdf that has mean equal to zero.