Homework # 3, Stat 526

1. The following table refers to an experiment on the use of sulfones and streptomycin drugs in the treatment of leprosy. The degree of infiltration at the start of the experiment measures a certain types of skin damage. The response refers to the change in the overall clinical condition of the patient after 48 weeks of the treatment. We use the equal-interval scores $\{1, 2, 3, 4, 5\}$. The question of interest is whether subjects with high infiltration changes differently from those with low infiltration.

	Degree of infiltration	
Clinical Change	High	Low
Worse	1	11
Stationary	13	53
Slight improvement	16	42
Moderate improvement	15	27
Marked improvement	7	11

   (a) Test the independence of the response and predictor based on Pearson χ^2 statistic (X^2) and likelihood-ratio statistic (G^2). What can you say about the significance of clinical change on the response.

   (b) Analyze these data using a logit model. Write down the fitted model and the significance of the parameters. Compute the odds ratio when the clinical change increases one level and compare the predicted proportion. What can you say about the significance of clinical change on the response.

   (c) Test independence using the Wald test. Is there an inconsistence for the significance of the slope in part (a) and (b). Comment on it.

2. Suppose a logit and probit model for Binary data with one predictor are fitted. Show that if the predictor variable is the negative value of the intercept over the slope then the predicted proportion is 1/2 (median). This value of the predictor is denoted by $LD50$. Find $LD50$ for complementary log-log link.

3. Show that the logistic regression function

$$\pi(x) = \frac{e^{\alpha+\beta x}}{1 + e^{\alpha+\beta x}}$$

   and the probit regression function

$$\pi(x) = \Phi(\alpha + \beta x)$$

have the steepest slopes at $\pi(x) = 0.5$. The complementary regression function

$$\pi(x) = 1 - e^{-e^{\alpha+\beta x}}$$

has the steepest slope at $\pi(x) = 1 - e^{-1}$.
Note: The steepest slopes tell us where the model is the most accurate for the fit.