1. Problem 8.9.

Rather than that provided by the textbook, we can also consider the test

\[ H_0 : \mu \leq 150 \leftrightarrow H_a : \mu > 150. \]

We need to reject \( H_0 \) if \( \bar{X} \geq c \) for some \( c \). We use test statistic \( \bar{X} \). The type I error probability is

\[ P(\bar{X} \geq c | \mu \leq 150) \]

and the type II error probability is

\[ P(\bar{X} < c | \mu > 150). \]

Both are functions of \( \mu \). The two approaches are equivalent in the computation of type I and type II error probabilities.

2. Problem 8.12.

The following is the solution for version 9.

(a) We test

\[ H_0 : \mu \leq 1300 \leftrightarrow H_a : \mu > 1300. \]

The mixture will be used if we conclude \( H_a \).

(b) We have \( \bar{X} \sim N(\mu, 60^2/10) = N(\mu, 360) \). We reject \( H_0 \) if \( \bar{X} \geq c \) for some \( c \). if \( \alpha = 0.01 \) is used, then \( c = 1300 + z_{0.01}\sqrt{360} = 1344 \). Therefore, we cannot reject \( H_0 \).

(c) The type II error probability is

\[ P(\bar{X} \leq 1344 | \mu = 1500) = \Phi\left(\frac{1344 - 1350}{\sqrt{360}}\right) = \Phi(0.21) = 0.3745. \]


(a) The rejection region is

\[ \frac{\bar{X} - 95}{1.20/\sqrt{16}} \geq z_{0.005} \Rightarrow C = \{ \bar{X} \geq 95.7728 \text{ or } \bar{X} \leq 94.2272 \}. \]

Because \( \bar{x} = 94.32 \not\in C \), \( H_0 \) is not rejected and thus we conclude that \( \mu = 95 \).

(b) The type II error is defined by

\[ P(\bar{X} \not\in C | \mu = 94) = P(94.2272 \leq \bar{X} \leq 95.7728 | \mu = 94) \]

\[ = \Phi\left(\frac{95.7728 - 94}{1.20/\sqrt{16}}\right) - \Phi\left(\frac{94.2272 - 94}{1.20/\sqrt{16}}\right) \]

\[ = 0.2244. \]

(c) Use the formula in Section 8.2. We have

\[ n = \left[ \frac{1.20(z_{0.005} + z_{0.1})}{95 - 94} \right]^2 = \left[ \frac{1.20(2.58 + 1.28)}{95 - 94} \right]^2 = 21.46 \Rightarrow n = 22. \]

We test

\[ H_0 : \mu = 15 \leftrightarrow \mu \neq 15. \]

The test statistic is

\[ z = \frac{\bar{x} - 15}{\sqrt{6.43^2/115}} = -6.17 \]

We reject \( H_0 \) if

\[ |z| > 1.96. \]

Therefore, we conclude the zinc intak does not fall between the recommended allowance.

5. Problem 8.35(a).

(a) We study the test

\[ H_0 : \mu \leq 200 \leftrightarrow H_A : \mu > 200. \]

The test statistic is

\[ T = \frac{\bar{x} - 200}{\sqrt{145.1/\sqrt{12}}} = 1.17, \]

which is lower than \( t_{0.05,11} = 1.80 \). Thus, we conclude that there is no compelling evidence.

6. Problem 8.45.

We have \( X \sim Bin(150, p) \). We test

\[ H_0 : p = 0.4 \leftrightarrow p \neq 0.4. \]

Then, \( \hat{p} = X/n = 82/150 = 0.5467 \). The test statistic is

\[ z = \frac{\hat{p} - 0.4}{\sqrt{0.4(1 - 0.4)/n}} = 3.6675. \]

We reject \( H_0 \) if \( |z| > z_{0.005} = 2.58 \). Therefore, we conclude \( H_a \) at \( \alpha = 0.01 \) significance level. If \( \alpha = 0.05 \) is used, we still conclude \( H_a \).