1. Problem 4.28.

(a) 
\[ P(0 \leq Z \leq 2.17) = \Phi(2.17) - \Phi(0) = 0.9850 - 0.5 = 0.485. \]

(b) 
\[ P(0 \leq Z \leq 1) = \Phi(1) - \Phi(0) = 0.8413 - 0.5 = 0.3413. \]

(c) 
\[ P(-2.50 \leq Z \leq 0) = \Phi(0) - \Phi(-2.50) = 0.5 - 0.0062 = 0.4938. \]

(d) 
\[ P(-2.50 \leq Z \leq 2.50) = \Phi(2.50) - \Phi(-2.50) = 0.9938 - 0.0062 = 0.9876. \]

(e) 
\[ P(Z \leq 1.37) = \Phi(1.37) = 0.9147. \]

(f) 
\[ P(-1.75 \leq Z) = 1 - \Phi(-1.75) = 1 - 0.0401 = 0.9599. \]

(g) 
\[ P(-1.50 \leq Z \leq 2.00) = \Phi(2.00) - \Phi(-1.50) = 0.9772 - 0.0668 = 0.9104. \]

(h) 
\[ P(1.37 \leq Z \leq 2.50) = \Phi(2.50) - \Phi(1.37) = 0.9938 - 0.9147 = 0.0791. \]

(i) 
\[ P(1.50 \leq Z) = 1 - \Phi(1.50) = 1 - 0.9332 = 0.0668. \]

(j) 
\[ P(1.37 \leq Z \leq 2.50) = 2\Phi(2.50) - 1 = 2(0.9938) - 1 = 0.9876. \]

2. Problem 4.29.

(a) 
\[ \Phi(c) = 0.9838 \Rightarrow c = 2.14. \]

(b) 
\[ P(0 \leq Z \leq c) = 0.291 \Rightarrow \Phi(c) = 0.791 \Rightarrow c = 0.81. \]

(c) 
\[ P(c \leq z) = 0.121 \Rightarrow \Phi(c) = 1 - 0.121 = 0.879 \Rightarrow c = 1.17. \]

(d) 
\[ P(-c \leq Z \leq c) = 0.668 \Rightarrow \Phi(c) - \Phi(-c) = 0.668 \]
\[ \Rightarrow \Phi(c) - [1 - \Phi(c)] = 0.668 \]
\[ \Rightarrow \Phi(c) = \frac{1 + 0.668}{2} = 0.834 \]
\[ \Rightarrow c = 0.97. \]

(a) Check the values from Table A.3.

\[ \Phi(c) = 0.91 \Rightarrow c \approx 1.34. \]

(b) Based on (a)

\[ \Phi(c) = 0.09 \Rightarrow \Phi(-c) = 0.91 \Rightarrow c \approx -1.34. \]

(c) Check the values from Table A.3.

\[ \Phi(c) = 0.75 \Rightarrow c \approx 0.675, \]

after take a look at the corresponding values of 0.67 and 0.68.

(d) Same reason as (b). The answer is −0.675.

(e)

\[ \Phi(c) = 0.06 \Rightarrow -1.555 \]

after take a look at the values of −1.55 and −1.56.


(a) \( z_{0.0055} = 2.54. \)

(b) \( z_{0.09} = 1.34. \) You may also write 1.35, but here 1.34 is much better than 1.35.

(c) \( z_{0.663} = -0.42. \) You may also write −0.43, but here −0.42 is much better than −0.43.

5. Problem 4.32.

We have the relationship of

\[ F(x) = P(X \leq x) = P\left( \frac{X - 15}{1.25} \leq \frac{x - 15}{1.25} \right) = \Phi\left( \frac{x - 15}{1.25} \right). \]

(a)

\[ P(X \leq 15) = \Phi(0) = 0.5. \]

(b)

\[ P(X \leq 17.5) = \Phi\left( \frac{17.5 - 15}{1.25} \right) = \Phi(2) = 0.9772. \]
(c) 
\[ P(X \geq 10) = 1 - P(X \leq 10) \]
\[ = 1 - \Phi\left(\frac{10 - 15}{1.25}\right) \]
\[ = 1 - \Phi(-4) \]
\[ = 1 - 0 \]
\[ = 1. \]

(d) 
\[ P(14 \leq X \leq 18) = \Phi\left(\frac{18 - 15}{1.25}\right) - \Phi\left(\frac{14 - 15}{1.25}\right) \]
\[ = \Phi(2.5) - \Phi(-0.8) \]
\[ = 0.9938 - 0.2119 \]
\[ = 0.7819. \]

(e) 
\[ P(|X - 15| \leq 3) = P(-3 \leq X - 15 \leq 3) \]
\[ = \Phi(12 \leq X \leq 18) \]
\[ = \Phi\left(\frac{18 - 15}{1.25}\right) - \Phi\left(\frac{12 - 15}{1.25}\right) \]
\[ = \Phi(2.5) - \Phi(-2.5) \]
\[ = 0.9938 - 0.0062 \]
\[ = 0.9876. \]

6. Problem 4.44.

(a) 
\[ P(-1.5\sigma \leq X - \mu \leq 1.5\sigma) = P(-1.5 \leq N(0,1) \leq 1.5) \]
\[ = 2\Phi(1.5) - 1 \]
\[ = 2 \times 0.9331 - 1 \]
\[ = 0.8664. \]

(b) 
\[ P(-2.5\sigma \leq X - \mu \leq 2.5\sigma) = 2\Phi(2.5) - 1 = 2 \times 0.9938 - 1 = 0.9876. \]

(c) 
\[ P(\sigma \leq |X - \mu| \leq 2\sigma) = |2\Phi(2) - 1| - |2\Phi(1) - 1| \]
\[ = 2|\Phi(2) - \Phi(1)| \]
\[ = 2(0.9772 - 0.8413) \]
\[ = 0.2718. \]

7. Problem 4.53.

When \( p = 0.5 \), \( E(X) = 25(0.5) = 12.5 \) and \( \sigma_x = \sqrt{25(0.5)(1 - 0.5)} = 2.5 \); when \( p = 0.6 \), \( E(X) = 25(0.6) = 15 \) and \( \sigma_x = \sqrt{25(0.4)(0.6)} = 2.449 \); when \( p = 0.8 \), \( E(X) = 20 \) and \( \sigma_x = 2 \). For normal approximation, we should take the interval \((14.5, 20.5)\) to compute the probability.
(a) When \( p = 0.5 \), by the normal approximation, we have
\[
P(15 \leq X \leq 20) \approx \Phi\left(\frac{20.5 - 12.5}{2.5}\right) - \Phi\left(\frac{14.5 - 12.5}{2.5}\right) \approx 0.9993 - 0.7881 = 0.2112;
\]
the exact probability is
\[
P(15 \leq X \leq 20) = P(X \leq 20) - P(X \leq 14) = 1.000 - 0.788 = 0.212.
\]
When \( p = 0.6 \), we have
\[
P(15 \leq X \leq 20) \approx \Phi\left(\frac{20.5 - 15}{2.449}\right) - \Phi\left(\frac{14.5 - 15}{2.449}\right) \approx 0.9876 - 0.4191 = 0.5685;
\]
the exact probability is
\[
P(15 \leq X \leq 20) = P(X \leq 20) - P(X \leq 14) = 0.991 - 0.414 = 0.577.
\]
When \( p = 0.8 \)
\[
P(15 \leq X \leq 20) \approx \Phi\left(\frac{20.5 - 20}{2}\right) - \Phi\left(\frac{14.5 - 20}{2}\right) \approx 0.5987 - 0.0028 = 0.5959;
\]
the exact probability is
\[
P(15 \leq X \leq 20) = P(X \leq 20) - P(X \leq 14) = 0.579 - 0.006 = 0.573.
\]

(b) Similarly, using normal approximation, we have
\[
P(X \leq 15) \approx \begin{cases} 
\Phi((15.5 - 12.5)/2.5) = 0.8850 & \text{when } p = 0.5 \\
\Phi((15.5 - 15)/2.449) = 0.5809 & \text{when } p = 0.6 \\
\Phi((15.5 - 20)/2) = 0.0122 & \text{when } p = 0.8
\end{cases}
\]
The exact probabilities are
\[
P(X \leq 15) = \begin{cases} 
0.8852 & \text{when } p = 0.5 \\
0.5754 & \text{when } p = 0.6 \\
0.0173 & \text{when } p = 0.8
\end{cases}
\]

(c) Using normal approximation, we have
\[
P(20 \leq X) = \begin{cases} 
1 - \Phi((19.5 - 12.5)/2.5) = 0.0026 & \text{when } p = 0.5 \\
1 - \Phi((19.5 - 15)/2.449) = 0.031 & \text{when } p = 0.6 \\
1 - \Phi((19.5 - 20)/2) = 0.5987 & \text{when } p = 0.8
\end{cases}
\]
The exact probabilities are
\[
P(X \leq 20) = \begin{cases} 
0.0021 & \text{when } p = 0.5 \\
0.0294 & \text{when } p = 0.6 \\
0.6167 & \text{when } p = 0.8
\end{cases}
\]