1. Problem 3.12.

(a) 
\[ P(\text{accommodate}) = P(Y \leq 50) = 0.05 + 0.10 + 0.12 + 0.14 + 0.25 + 0.17 = 0.83. \]

(b) 
\[ P(Y > 50) = 10P(Y \leq 5) = 0.17. \]

(c) 
\[ P(\text{the first standby can be accommodated}) = P(Y \leq 49) = 0.05 + 0.10 + 0.12 + 0.14 + 0.25 = 0.66 \]

and 
\[ P(\text{the third standby can be accommodated}) = P(Y \leq 47) = 0.05 + 0.10 + 0.12 = 0.27. \]


(a) The probability of at most 3 lines in use is 
\[ P(X \leq 3) = p(0) + p(1) + p(2) + p(3) = 0.1 + 0.15 + 0.20 + 0.25 = 0.70. \]

(b) The probability of fewer than 3 lines in use is 
\[ P(X < 3) = P(X \leq 2) = p(0) + p(1) + p(2) = 0.1 + 0.15 + 0.20 = 0.45. \]

(c) The probability of at least 3 lines in use is 
\[ P(X \geq 3) = p(3) + p(4) + p(5) + p(6) = 0.25 + 0.20 + 0.06 + 0.04 = 0.55; \]

or 
\[ P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.45 = 0.55. \]

(d) The probability of between 2 and 5 lines, inclusive, in use is 
\[ P(2 \leq X \leq 5) = p(2) + p(3) + p(4) + p(5) = 0.20 + 0.25 + 0.20 + 0.06 = 0.71. \]

(e) The probability of between 2 and 4 lines, inclusive, not in use is the probability of between 6 - 2 = 4 and 6 - 2 = 4 in use, which is 
\[ P(2 \leq X \leq 4) = p(2) + p(3) + p(4) = 0.20 + 0.25 + 0.20 = 0.65. \]

(f) At least 4 lines not in use means at most 2 lines in use. Thus, the probability is 
\[ P(X \leq 2) = 0.45. \]

3. Problem 3.14. It is known that \( P(Y = y) = ky \) for \( k = 1, 2, 3, 4, 5. \)
(a) Then, we have
\[
\sum_{y=1}^{5} P(Y = y) = \sum_{y=1}^{5} ky = 15k = 1 \Rightarrow k = \frac{1}{15}.
\]

(b) The probability is
\[
P(Y \leq 3) = p(1) + p(2) + p(3) = \frac{1 + 2 + 3}{15} = 0.4.
\]

(c) The probability is
\[
P(2 \leq Y \leq 4) = \frac{2 + 3 + 4}{15} = 0.6.
\]

(d) It is clear that \(y^2/50 > 0\) for \(y = 1, 2, 3, 4, 5\). Because
\[
\sum_{y=1}^{5} \frac{y^2}{50} = \frac{55}{50} \neq 1,
\]

it could not be the pmf of \(Y\).


(a) If \(Y = 2\), then all the two must be acceptable. Thus

\[
p(2) = P(Y = 2) = 0.9^2 = 0.81.
\]

(b) If \(Y = 3\), then exactly one of the first two is acceptable and the third is acceptable. Thus,

\[
P(3) = \binom{2}{0} 0.9^2 \times 0.1 = 0.162.
\]

(c) If \(Y = 5\), then exactly one of the first four is acceptable and the fifth is acceptable. Thus,

\[
p(5) = P(AUUUA) + P(UAUUA) + P(UUAA) + P(UUUAA)
\]
\[
= 4 \times 0.9^2 \times 0.1^3
\]
\[
= 0.00324.
\]

(d) In general, we have
\[
p(y) = \binom{y - 1}{1} 0.9^2 0.1^{y-2} = (y - 1) 0.9^2 0.1^{y-2}
\]
for \(y = 2, 3, 4 \cdots\).

5. Problem 3.18.

(a) The PMF of \(M\) is

<table>
<thead>
<tr>
<th>(m)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p(m))</td>
<td>(\frac{1}{36})</td>
<td>(\frac{3}{36})</td>
<td>(\frac{5}{36})</td>
<td>(\frac{7}{36})</td>
<td>(\frac{9}{36})</td>
<td>(\frac{11}{36})</td>
</tr>
</tbody>
</table>
(b) The CDF of \( M \) is

\[
F(m) = \begin{cases}
0 & m < 1 \\
\frac{1}{36} & 1 \leq m < 2 \\
\frac{4}{36} & 2 \leq m < 3 \\
\frac{9}{36} & 3 \leq m < 4 \\
\frac{15}{36} & 4 \leq m < 5 \\
\frac{25}{36} & 5 \leq m < 6 \\
1 & m \geq 6
\end{cases}
\]


(a) 

\[
P(X = 2) = F(2) - F(2^-) = F(2) - F(1) = 0.39 - 0.19 = 0.20.
\]

(b) 

\[
P(X > 3) = 1 - P(X \leq 3) = 1 - F(3) = 1 - 0.67 = 0.33.
\]

(c) 

\[
P(2 \leq X \leq 5) = P(X \leq 5) - P(X < 2) = F(5) - F(2^-) = F(5) - F(1) = 0.97 - 0.19 = 0.78.
\]

(d) 

\[
P(2 < X < 5) = P(X < 5) - P(X \leq 2) = F(5) - F(2) = F(4) - F(2) = 0.92 - 0.39 = 0.53.
\]


(a) The PMF are

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(x) )</td>
<td>0.30</td>
<td>0.10</td>
<td>0.05</td>
<td>0.15</td>
<td>0.40</td>
</tr>
</tbody>
</table>

(b) 

\[
P(3 \leq X \leq 6) = P(X \leq 6) - P(X < 3) = F(6) - F(2) = 0.6 - 0.3 = 0.3.
\]

\[
P(4 \leq X) = 1 - P(X < 4) = 1 - F(3) = 1 - 0.4 = 0.6.
\]