1. Problem 2.77.

(a) Let $p$ be the probability of defect. Then,

$$
(1 - p)^{25} = 0.85 \Rightarrow p = 0.0065
$$

(b)

$$
(1 - p)^{25} \geq 0.9 \Rightarrow p \leq 0.0042.
$$

2. Problem 2.78. Let $A_i$ be the event that valve $i$ opens. The event “at least one valve open” is the complement of “none open”. Therefore, the probability of “at least one valve open” is

$$
1 - P(A_1' \cap A_2' \cap A_3' \cap A_4' \cap A_5') = 1 - \sum_{i=1}^{5} P(A_i') = 1 - (1 - 0.96)^5 = 0.9999999 \approx 1.
$$

The event “at least one valve fails to open” is the complement of “none of them fails to open”. Thus, it is

$$
1 - P(\bigcap_{i=1}^{5} A_i) = 1 - 0.96^5 = 0.1846.
$$

3. Problem 2.80.

$$
P(\text{system works})
$$

$$
= P((A_1 \cup A_2) \cup (A_3 \cap A_4))
$$

$$
= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P((A_1 \cup A_2) \cap (A_3 \cap A_4))
$$

$$
=[P(A_1) + P(A_2) - P(A_1 \cap A_2)] + P(A_3)P(A_4) - [P(A_1) + P(A_2) - P(A_1 \cap A_2)]P(A_3)P(A_4)
$$

$$
=(0.9 + 0.9 - 0.9^2) + 0.8^2 - (0.9 + 0.9 - 0.9^2)(0.8^2)
$$

$$
= 0.9964.
$$

4. Problem 2.82. By the counting method, we have $P(A) = P(B) = 1/6$, $P(A \cap B) = P(A \cap C) = P(B \cap C) = P(A \cap B \cap C) = 1/36$. Since $P(A \cap B) = P(A)P(B)$, we conclude that $A$ and $B$ are independent. Similarly, we conclude that $A$ and $C$, and $B$ and $C$ are independent. Since $P(A \cap B \cap C) \neq P(A)P(B)P(C)$, we conclude that $A$, $B$, and $C$ are not mutually independent.

5. Problem 2.84.

(a)

$$
P(A_1 \cap A_2 \cap A_3) = 0.95 \times 0.98 \times 0.80 = 0.7448.
$$

(b)

$$
1 - P(A_1 \cap A_2 \cap A_3) = 1 - 0.7448 = 0.2552.
$$

(c)

$$
P(A_1' \cap A_2' \cap A_3') = (1 - 0.95)(1 - 0.98)(1 - 0.80) = 0.0002.
$$

(d)

$$
P(\text{Exactly one}) = P(A_1 \cap A_2' \cap A_3') + P(A_1' \cap A_2 \cap A_3') + P(A_1' \cap A_2' \cap A_3)
$$

$$
= 0.95(1 - 0.98)(1 - 0.80)
$$

$$
= 0.038.
$$
(e) 

\[ P(A_1 \cap A'_2 \cap A'_3) + P(A'_1 \cap A_2 \cap A'_3) + P(A'_1 \cap A'_2 \cap A_3) = 0.95(1 - 0.98)(1 - 0.80) + (1 - 0.95)0.98(1 - 0.80) + (1 - 0.95)(1 - 0.98)0.8 = 0.0144. \]

(f) This cannot be computed by the conditions.

6. Problem 2.87.

(a) 

\[ P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = 0.55 + 0.65 - 0.80 = 0.4. \]

(b) 

\[ P(A_2 | A_3) = \frac{P(A_2 \cap A_3)}{P(A_3)} = \frac{0.40}{0.70} = 0.5714. \]

It describes the conditional probability of that the person likes the second vehicle given that the person likes the second vehicle.

(c) \( A_2 \) and \( A_3 \) are not independent because \( P(A_2 | A_3) \neq P(A_2) \) or \( P(A_1 \cap A_3) \neq P(A_2)P(A_3) \).

(d) 

\[
P(A_2 \cup A_3 | A'_1) = \frac{P((A_2 \cup A_3) \cap A'_1)}{P(A'_1)} = \frac{P(A_1 \cup A_2 \cup A_3) - P(A_1)}{P(A'_1)} = \frac{0.88 - 0.55}{0.45} = 0.7333. 
\]

Note: The problem contains an error. It has \( P(A_2 \cap A_3) = 0.4 + 0.65 - 0.7 = 0.95 \geq P(A_1 \cup A_2 \cup A_3) = 0.99 \). However, this does not affect the results.

7. Problem 2.88. Let \( D \) be the disease and \( A \) be the test to be positive. Then, \( P(D) = 0.05 \), \( P(A | D) = 0.98 \), and \( P(A' | D') = 0.99 \). Let \( A_1 \) and \( A_2 \) be positive result in the first and the second times. Then, \( P(A_1 \cap A_2 | D) = 0.98^2 = 0.9604 \) and \( P(A_1 \cap A_2 | D') = 0.01^2 = 0.0001 \). By Bayes’s Theorem, we have

\[
P(D | A_1 \cap A_2) = \frac{P(A_1 \cap A_2 | D) P(D)}{P(A_1 \cap A_2 | D) P(D) + P(A_1 \cap A_2 | D') P(D')} = \frac{0.9604 \times 0.05}{0.9604 \times 0.05 + 0.0001 \times 0.95} = 0.9980. \]