1. Problem 2.77.

(a) Let \( p \) be the probability of defect. Then,

\[
(1 - p)^{25} = 0.85 \Rightarrow p = 0.0065
\]

(b)

\[
(1 - p)^{25} \geq 0.9 \Rightarrow p \leq 0.0042.
\]

2. Problem 2.78. Let \( A_i \) be the event that valve \( i \) opens. The event “at least one valve open” is the complement of “none open”. Therefore, the probability of “at least one valve open” is

\[
1 - P(\bigcap_{i=1}^{5} A_i) = 1 - \left(1 - 0.96\right)^5 = 0.9999999 \approx 1.
\]

The event “at least one vales fails to open” is the complement of “none of them fails to open”. Thus, it is

\[
1 - P(\bigcap_{i=1}^{5} A_i) = 1 - 0.96^5 = 0.1846.
\]

3. Problem 2.80.

\[
P(\text{system works}) = P((A_1 \cup A_2) \cup (A_3 \cap A_4))
\]

\[
= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P((A_1 \cup A_2) \cap (A_3 \cap A_4))
\]

\[
= [P(A_1) + P(A_2) - P(A_1 \cap A_2)] + P(A_3)P(A_4) - [P(A_1) + P(A_2) - P(A_1 \cap A_2)]P(A_3)P(A_4)
\]

\[
= (0.9 + 0.9 - 0.9^2) + 0.8^2 - (0.9 + 0.9 - 0.9^2)0.8^2
\]

\[
= 0.9964.
\]

4. Problem 2.82. By the counting method, we have \( P(A) = P(B) = 1/6 \), \( P(A \cap B) = P(A \cap C) = P(B \cap C) = P(A \cap B \cap C) = 1/36 \). Since \( P(A \cap B) = P(A)P(B) \), we conclude that \( A \) and \( B \) are independent. Similarly, we conclude that \( A \) and \( C \), and \( B \) and \( C \) are independent. Since \( P(A \cap B \cap C) \neq P(A)P(B)P(C) \), we conclude that \( A, B, \) and \( C \) are not mutually independent.

5. Problem 2.84.

(a)

\[
P(A_1 \cap A_2 \cap A_3) = 0.95 \times 0.98 \times 0.80 = 0.7448.
\]

(b)

\[
1 - P(A_1 \cap A_2 \cap A_3) = 1 - 0.7448 = 0.2552.
\]

(c)

\[
P(A_1' \cap A_2' \cap A_3') = (1 - 0.95)(1 - 0.95)(1 - 0.80) = 0.0002.
\]

(d)

\[
P(A_1 \cap A_2' \cap A_3') = 0.95(1 - 0.95)(1 - 0.80) = 0.0038.
\]
(e) 
\[
P(A_1 \cap A'_2 \cap A'_3) + P(A'_1 \cap A_2 \cap A'_3) + P(A'_1 \cap A'_2 \cap A_3)
\]
\[
= 0.95(1 - 0.95)(1 - 0.80) + 0.95(1 - 0.95)(1 - 0.80) + 0.80(1 - 0.95)(1 - 0.95)
\]
\[
= 0.021.
\]

(f) This cannot be computed by the conditions.

6. Problem 2.87.

(a) 
\[
P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = 0.55 + 0.65 - 0.80 = 0.4.
\]

(b) 
\[
P(A_2|A_3) = \frac{P(A_2 \cap A_3)}{P(A_3)} = \frac{0.40}{0.70} = 0.5714.
\]

It describes the conditional probability of that the person likes the second vehicle given that the person likes the second vehicle.

(c) \(A_2\) and \(A_3\) are not independent because \(P(A_2|A_3) \neq P(A_2)\) or \(P(A_1 \cap A_3) \neq P(A_2)P(A_3)\).

(d) 
\[
P(A_2 \cup A_3|A'_1) = \frac{P((A_2 \cup A_3) \cap A'_1)}{P(A'_1)}
\]
\[
= \frac{P(A_1 \cup A_2 \cup A_3) - P(A_1)}{P(A'_1)}
\]
\[
= \frac{0.88 - 0.55}{0.45}
\]
\[
= 0.7333.
\]

Note: The problem contains an error. It has \(P(A_2 \cap A_3) = 0.4 + 0.65 - 0.7 = 0.95 \geq P(A_1 \cup A_2 \cup A_3) = 0.99\). However, this does not affect the results.

7. Problem 2.88. Let \(D\) be the disease and \(A\) be the test to be positive. Then, \(P(D) = 0.05\), \(P(A|D) = 0.98\), and \(P(A'|D') = 0.99\). Let \(A_1\) and \(A_2\) be positive result in the first and the second times. Then, \(P(A_1 \cap A_2|D) = 0.98^2 = 0.9604\) and \(P(A_1 \cap A_2|D') = 0.01^2 = 0.0001\). By Bayes’s Theorem, we have

\[
P(D|A_1 \cap A_2) = \frac{P(A_1 \cap A_2|D)P(D)}{P(A_1 \cap A_2|D)P(D) + P(A_1 \cap A_2|D')P(D')}
\]
\[
= \frac{0.9604 \times 0.05}{0.9604 \times 0.05 + 0.0001 \times 0.95}
\]
\[
= 0.9980.
\]