1. Problem 2.13.

(a) \[ P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.22 + 0.25 - 0.11 = 0.36. \]

(b) \[ P(A_1' \cap A_2') = P((A_1 \cup A_2)') = 1 - P(A_1 \cup A_2) = 1 - 0.36 = 0.64. \]

(c) \[ P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_2 \cap A_3) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3) \]
\[ = 0.22 + 0.25 + 0.28 - 0.11 - 0.05 - 0.07 + 0.01 = 0.53. \]

(d) \[ P(A_1' \cap A_2' \cap A_3') = P((A_1 \cup A_2 \cup A_3)') = 1 - P(A_1 \cup A_2 \cup A - 3) \]
\[ = 1 - 0.53 = 0.47. \]

(e) \[ P(A_1' \cap A_2' \cap A_3') = P(A_1' \cap A_2') - P(A_1' \cap A_2' \cap A_3') \]
\[ = 0.64 - 0.47 = 0.17. \]

(f) \[ P((A_1' \cap A_2') \cup A_3') = P(A_1' \cap A_2' \cap A) + P(A_3') - P(A_1' \cap A_2' \cap A_3') \]
\[ = 0.64 + 0.28 - 0.17 = 0.75. \]

2. Problem 2.17.

(a) Because \((A \cup B)'\) is not empty.

(b) \(P(A') = 1 - P(A) = 0.7.\)

(c) Since \(A \cap B = \phi, P(A \cup B) = P(A) + P(B) = 0.8.\)

(d) \(P(A' \cap B') = 1 - P(A \cup B) = 0.2.\)

3. Problem 2.21.

(a) \(P(\{\text{Auto High}\} \cap \{\text{Homeowner High}\}) = 0.10.\)

(b) \(P(\{\text{Auto Low}\}) = 0.04 + 0.06 + 0.05 + 0.03 = 0.18; P(\{\text{Homeowner Low}\}) = 0.06 + 0.10 + 0.03 = 0.19.\)

(c) \(P(\{\text{Same Categories}\}) = 0.06 + 0.20 + 0.15 = 0.41.\)

(d) \(P(\{\text{Different Categories}\}) = 1 - 0.41 = 0.59.\)

(e) \(P(\{\text{At Least One is Low}\}) = 0.04 + 0.06 + 0.05 + 0.03 + 0.10 + 0.03 = 0.31.\)

(f) \(P(\{\text{Neither is Low}\}) = 1 - 0.31 = 0.69.\)

4. Problem 2.22. Let \(A\) and \(B\) be the probability that he must stop at the first and second, respectively. Then, \(P(A) = 0.4, P(B) = 0.5,\) and \(P(A \cup B) = 0.7.\)
(a) \[ P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.4 + 0.5 - 0.7 = 0.2. \]

(b) \[ P(A \cap B') = P(A) - P(A \cap B) = 0.4 - 0.2 = 0.2. \]

(c) \[ P(A \cap B') + P(A' \cap B) = 0.2 + P(B) - P(A \cap B) = 0.2 + 0.5 - 0.2 = 0.5. \]

5. Problem 2.24. If \( A \subseteq B \), then \( B = A \cup (B \cap A') \) and \( A \) and \( B \cap A' \) are disjoint. Thus,

\[ P(B) = P(A) + P(B \cap A') \geq P(A) \]

by the axioms 3a and 1. For general \( A \) and \( B \), it means

\[ P(A \cap B) \leq P(A) \leq P(A \cup B). \]

6. Problem 2.25. It tells us that \( P(A) = 0.4, P(B) = 0.55, P(C) = 0.7, P(A \cup B) = 0.63, P(A \cup C) = 0.77, P(B \cup C) = 0.8 \) and \( P(A \cup B \cup C) = 0.85 \).

(a) It is exactly

\[ P(A \cup B \cup C) = 0.85 \]

as it is given.

(b) None of selected is the complementary of at least one selected. Thus it is

\[ P((A \cup B \cup C)') = 1 - P(A \cup B \cup C) = 1 - 0.85 = 0.15. \]

(c) Only automatic means \( C \) but not \( A \) and \( B \). Thus

\[ P(A' \cap B' \cap C') = P(A' \cap B') - P(A' \cap B' \cap C') \]
\[ = [1 - P(A \cup B)] - [1 - P(A \cup B \cup C)] \]
\[ = P(A \cup B \cup C) - P(A \cup B) \]
\[ = 0.85 - 0.63 = 0.22. \]

(d) Exactly one of the three is

\[ P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C) \]
\[ = [P(A \cup B \cup C) - P(B \cup C)] + [P(A \cup B \cup C) - P(A \cup C)] + [P(A \cup B \cup C) - P(A \cup B)] \]
\[ = 0.05 + 0.08 + 0.22 = 0.35. \]


(a) The probability that the system does not have a type 1 defect is

\[ P(A'_1) = 1 - P(A_1) = 1 - 0.12 = 0.88. \]

(b) The probability that the system has both type 1 and type 2 defects is

\[ P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = 0.12 + 0.07 - 0.13 = 0.06. \]
(c) The probability that the system has both type 1 and type 2 but not type 3 is

\[ P(A_1 \cap A_2 \cap A_3') = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) = 0.06 - 0.01 = 0.05. \]

(d) The event that the system has at most two of these defects is the complement of the event that the system has all of the three defects. Thus, it is

\[ 1 - P(A_1 \cap A_2 \cap A_3) = 1 - 0.01 = 0.99. \]