There are totally 35 points in the exam. Please write down your name, student ID number below.

NAME: __________________________________________
ID: __________________________________________

Section 11: ___ 8:30-9:20MWF
Section 3: ___ 9:30-10:20MWF
1. (15 points, 1 point each). Fill in the blanks. Your answer should be real numbers.

(a) (3 points). Let $A$ and $B$, and $C$ be events. Assume that $P(A) = 0.28$, $P(B) = 0.36$, and $P(A \cap B) = 0.19$. Then, $P(A') = \underline{\hspace{2cm}}$, $P(A \cup B) = \underline{\hspace{2cm}}$, and $P(A|B) = \underline{\hspace{2cm}}$.

(b) (3 points). Let $A$ and $B$ be events. Assume that $P(A) = 0.55$, $P(B|A) = 0.36$, and $P(B|\bar{A}) = 0.84$. Then, $P(A \cap B) = \underline{\hspace{2cm}}$, $P(B) = \underline{\hspace{2cm}}$, and $P(A|B) = \underline{\hspace{2cm}}$.

(c) (3 points). Let $A$, $B$, and $C$ be independent events with $P(A) = 0.3$, $P(B) = 0.6$, and $P(C) = 0.75$. Then, $P(A' \cap B \cap C') = \underline{\hspace{2cm}}$, $P(A \cup B \cup C) = \underline{\hspace{2cm}}$, and $P(A|B \cup C) = \underline{\hspace{2cm}}$.

(d) (4 points). Flip a balanced die 5 times. Let $X$ be the total number of 5 or 6. Then, $P(X \leq 1) = \underline{\hspace{2cm}}$, $E(X) = \underline{\hspace{2cm}}$, and $V(X) = \underline{\hspace{2cm}}$.

(e) (4 points). Suppose $X \sim N(3.7, 1.28)$. Let $Y = -0.5X + 1.4$. Then, $P(X \leq 5.0) = \underline{\hspace{2cm}}$, $E(Y) = \underline{\hspace{2cm}}$, and $V(Y) = \underline{\hspace{2cm}}$. 
2. (8 points). Suppose the joint PMF of $X$ and $Y$ is

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>0.04</td>
<td>0.09</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>1</td>
<td>0.07</td>
<td>0.11</td>
<td>0.08</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
<td>0.10</td>
<td>0.05</td>
<td>0.05</td>
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(a) (2 points). Compute the marginal PMFs of $X$ and $Y$, respectively.

(b) (2 points). Compute $E(X)$, $E(Y)$, $V(X)$, and $V(Y)$.

(c) (2 points). Compute $\text{Cov}(X,Y)$ and $\text{Corr}(X,Y)$.

(d) (2 points). Are $X$ and $Y$ independent. Justify your answer.
3. (6 points). Use the Central Limit Theorem (CLT) to compute the following probabilities. You do not need to consider 0.5-shift.

(a) (2 points). Flip a balanced coin 5000 times. Let $X$ be the total number of heads. Compute $P(2400 \leq X \leq 2600)$.

(b) (2 points). Let $X_1, \ldots, X_n$ be identically and independently observed with the common expected value $\mu = E(X_i) = 0.7$ and the common variance $\sigma^2 = V(X_i) = 1.9$. Compute $P(0.69 \leq \bar{X} \leq 0.71)$ for $n = 10^3$, $10^4$, and $10^5$, respectively.

(c) (2 points). Let $X_1, \ldots, X_{1000}$ be identically and independent observed with the common PMF given by

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
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</thead>
<tbody>
<tr>
<td>$p(x)$</td>
<td>0.25</td>
<td>0.15</td>
<td>0.2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Compute $P(2.70 \leq \bar{X} \leq 2.80)$. 

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4. (6 points). Let $X_1 \sim N(0.7, 1.4)$, $X_2 \sim N(-0.5, 1.2)$, $X_3 \sim N(0.9, 1.7)$, and $X_4 \sim N(-0.2, 2.1)$, independently.

(a) (2 points). Compute $P(-0.5 X_1 - X_2 + 1.5 X_3 - 1.2 X_4 \leq 5)$.

(b) (2 points). Compute $P(|2.5 X_1 + 0.5 X_2 - 2.2 X_3 - 1.8 X_4 - 2| \leq 4)$.

(c) (2 points). Compute $P(2 X_1 - 1.2 X_2 \leq 3.1 X_3 - 1.4 X_4 + 9)$. 