Section 7.4: Confidence Interval for Variance and Standard Deviation of a Normal Population
Suppose $X_1, \cdots, X_n$ are iid observations of a normal sample, say $N(\mu, \sigma^2)$. Then,
\[
\frac{(n - 1)S^2}{\sigma^2} \sim \chi^2_{n-1}.
\]
Thus, we have
\[
P(\chi^2_{1 - \alpha/2,n-1} \leq \frac{(n - 1)S^2}{\sigma^2} \leq \chi^2_{\alpha/2,n-1}) = 1 - \alpha
\]
which gives the $1 - \alpha$ level confidence interval for $\sigma^2$ as
\[
\left[ \frac{(n - 1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n - 1)s^2}{\chi^2_{1 - \alpha/2,n-1}} \right].
\]
and for $\sigma$ as
\[
\left[ \sqrt{\frac{(n - 1)s^2}{\chi^2_{\alpha/2,n-1}}}, \sqrt{\frac{(n - 1)s^2}{\chi^2_{1 - \alpha/2,n-1}}} \right].
\]
First example of Section 7.4: example 7.15 on textbook. In this example, the data collected 17 observations of breakdown voltage. We have \( n = 17 \) and \( s^2 = 137324.3 \). Thus for 95% confidence interval for \( \sigma^2 \), we need
\[
\chi^2_{0.975,16} = 6.908
\]
and
\[
\chi^2_{0.025,16} = 28.845.
\]
Thus, the 95% confidence interval for \( \sigma^2 \) is
\[
\left[ \frac{16s^2}{28.845}, \frac{16s^2}{6.908} \right] = [76172.3, 318064.4]
\]
and the 95% confidence interval for \( \sigma \) is
\[
[\sqrt{76172.3}, \sqrt{318064.4}] = [276.0, 564.0].
\]