Homework # 3, Stat355, Spring 2021

Please use R to do all of the following problems. Submit your homework by a WORD or PDF document with main results and the corresponding code.

1. Use matrix methods in this problems.

   (a) Find one solution of the following linear system by the generalized inverse and verify your solution.

\[
\begin{align*}
2x_1 + x_2 + 2x_3 + 3x_4 &= 13 \\
x_1 - 2x_2 + x_3 + x_4 &= 8 \\
3x_1 + x_2 + x_3 - x_4 &= 1.
\end{align*}
\]

   (b) Using both the inverse and the generalized inverse to solve the following linear system. Verify your solutions whether they are equal.

\[
\begin{align*}
x_1 + x_2 + x_3 &= 1 \\
x_1 + x_2 - 2x_3 &= 3 \\
2x_1 + x_2 + x_3 &= 2.
\end{align*}
\]

2. The following table reports the population sizes (in millions) of three European countries from 1990 to 2020.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>57.16</td>
<td>57.94</td>
<td>58.79</td>
<td>60.18</td>
<td>62.51</td>
<td>64.85</td>
<td>67.89</td>
</tr>
<tr>
<td>France</td>
<td>58.24</td>
<td>59.54</td>
<td>60.91</td>
<td>63.18</td>
<td>65.03</td>
<td>66.55</td>
<td>67.13</td>
</tr>
<tr>
<td>Germany</td>
<td>62.68</td>
<td>81.54</td>
<td>82.16</td>
<td>82.50</td>
<td>81.79</td>
<td>81.21</td>
<td>83.02</td>
</tr>
</tbody>
</table>

   (a) Get a bar plot for the population sizes for the three countries. You need to use different colors to represent the three counties with legend for the interpretation of the colors.

   (b) Get a bar plot for the changes of the population sizes for each of the countries (\(\text{pop} - \text{pop}_{1990}\)). You also need to use different colors to represent the changes with a legend for the interpretation of the colors. Provide a reason for why there is a sudden increase in Germany from 1990 to 1995.

3. Combine two images files. You can use your smart phone to get two jpg files and load them to R. You need to get a patch from the first image and put in somewhere in the second image. Then, write it to the hard disk. You will have a combined image. Please provide the two original images and the combined images with your R code. You need interpret what you have done. To avoid human subject issues, please do not include any human faces in these images.