Regression PCA for Moving Objects Separation

Huyunting Huang, Xiang Liu, Tonglin Zhang, and Baijian Yang
Purdue University
West Lafayette, Indiana
{ huan1182, xiang35, tlzhang, byang}@purdue.edu

***Abstract*—This work proposed a new approach called regression PCA (RegPCA) for statistical machine learning and big data analyses. One of the potential use cases investigated in this work is to separate the moving objects (foreground) from the background images. This is achieved by performing regression before conducting Robust PCA (RPCA). RegPCA works well in the moving object detection task because the background information can be conceived as the regression portion of the images, while the residual portion of the regression can then be fed into RPCA to fine tune the foreground detection. The experiments show that in moving object detection problems RegPCA provides much better results than applying only RPCA, especially in color videos and when the moving objects are relatively big. Further studies are needed to leverage the interesting features of RegPCA approach and apply it to solve more real world problems.**

***Index Terms*—PCA, Regression, RPCA, Big Data, foreground detection**

I. Introduction

Principal Component Analysis (PCA) and linear regression are two well known data analysis methods which have been widely applied in many areas, such as statistical machine learning and big data analyses. The two methods are often complimentary to each other and can be used hand-in-hand, as in the principal component regression (PCR) approach [1]–[5]. The philosophy is that PCA can extract the most important information from explanatory variables and only those principle components of the explanatory variables will be used as regressors. Because the dimension of the explanatory variables are reduced, the speed and the quality of data analyses are often improved.

The idea of RegPCA is motivated from PCR but in reverse order. It applies regression before employing PCA. The regression step accounts for explanatory variables. By treating a variable of interest as the response, the impact of explanatory variables can be removed by a regression model. Then, the PCA step is applied. This step implements the traditional or a modified PCA to the residuals of the regression model. An obvious advantage is that RegPCA can remove redundant information in traditional PCA for dimension reduction or RPCA for moving object detection. Theoretically, the idea of RegPCA can be used to any PCA approaches and its variants. This is because the two steps are implemented sequentially. RegPCA can be easily combined with any modified PCA and the implementation is flexible. If the regression step is ignored, then it is the PCA method. If the PCA step is ignored, then it is a pure regression method. When both are applied, we call it a RegPCA method.

We focus on RPCA [6] in this paper because RPCA is one of the most significant modifications for high-dimensional image and video analysis. Some successful RPCA applications include Dynamic Mode Decomposition to separate background and foreground in greyscale videos [7].

RPCA is powerful in decomposing data matrix for videos into a low rank component and a sparse component. If the light intensity of background in a video is stable, then the background information can be represented by the low rank component and the moving object information can be represented by the sparse component of the data matrix. This property leads to video separation for foreground and background. Examples include the Grassmannian Robust Adaptive Subspace Tracking Algorithm (GRASTA) [8] and Panoramic Robust PCA [9]. Although powerful in greyscale videos, RPCA finds itself fragile in color videos. When both the shapes and the colors of the objects must be considered, RPCA does not separate them well. Although RPCA can put moving objects into sparse component, it does not provide correct color information. If the moving objects are not small, then results given by RPCA are usually meaningless. The proposed RegPCA method is an interesting answer to address this challenge. In video processing, RegPCA expresses a given video into a regression term for its background and an error term for moving objects. In general, RegPCA aims to maintain most background information in the regression term and keep most information of moving objects into model residuals.

In our experiments, we implemented RegRPCA with comparison to RPCA for video separation. We consider three scenarios for evaluation. The first scenario is an ideal case: the camera is at a fixed position; the light intensity does not change over time; and the moving objects are small. The second scenario assumes image stabilization is not available at neither the hardware nor the software level. There are jitters in videos; there are lighting intensity changes; and the moving object is sparse. In the third scenario, we consider a more realistic case when the moving objects are not small and therefore the residual matrix is not sparse. We assume the camera is mounted at a fixed position and the lighting condition does not change significantly over time. We also developed a metric called *color deviation* to evaluate how much color information are correctly retained after the foreground and background separation task.

The contributions of this paper are:

- We proposed RegPCA, a new way to combine Regression and PCA method.
- We combined RegPCA with RPCA with implementation as an example to show the ease of modification of RegPCA.

- We applied RegRPCA in color video separation task to show a potential application of the method.

The remainder of the paper is structured as follows. In Section II, relevant background is introduced. In Section III, the method of RegPCA, RPCA, combination of two and implementation in video separation are introduced. Experiments and analysis are described and discussed in Section IV and the paper is concluded in Section V.

II. Background

Traditional principal component analysis (PCA) was first invented by Karl Pearson in 1901 as an analogue of the principal axis theorem in mechanics [10]. The method was independently developed by Harold Hotelling in 1930s [11]. The goal of PCA is to find a subspace of the highly correlated high-dimensional data matrix with lower dimensional linear combinations, such that the result can maintain the most variations of the original data matrix.

Let $\mathbf{M}$ be an $n \times p$ matrix. By singular value decomposition (SVD) to $\mathbf{M}$, it is expressed as

$$\mathbf{M} = \mathbf{U}\mathbf{D}\mathbf{V}^\top \tag{1}$$

where $\mathbf{U} = (\mathbf{u}_1, \dots, \mathbf{u}_p)$ is a $n \times p$ orthogonal matrix satisfying $\mathbf{U}^\top\mathbf{U} = \mathbf{I}_p$, $\mathbf{V} = (\mathbf{v}_1, \dots, \mathbf{v}_p)$ is a $p \times p$ orthogonal matrix for loadings satisfying$\mathbf{V}^\top\mathbf{V} = \mathbf{I}_p$, and $\mathbf{D} = \mathrm{diag}(d_1, \dots, d_p)$ is a $p \times p$ diagonal matrix for singular values. The singular values are assumed to be ordered such that $d_p \geq d_2 \geq, \dots, \geq d_p \geq 0$. The columns of $\mathbf{Z} = \mathbf{U}\mathbf{D}$ are the principal components (PCs) and the columns of $\mathbf{V}$ are the corresponding loadings. The i th PC is $\mathbf{PC}_i = d_i\mathbf{u}_i$ and its sample variance is d_i^2/n. For any integer $k \leq p$, let

$$\mathbf{M}_k = \sum_{i=1}^{k} d_i \mathbf{u}_i \mathbf{v}_i^\top = \mathbf{U}_k \mathbf{D}_k \mathbf{V}_k^\top, \tag{2}$$

where $\mathbf{U}_k = (\mathbf{u}_1, \dots, \mathbf{u}_k)$, $\mathbf{D}_k = \mathrm{diag}(d_1, \dots, d_k)$, and $\mathbf{V}_k = (\mathbf{v}_1, \dots, \mathbf{v}_k)$. The variation of $\mathbf{M}_k$ is $\sum_{i=1}^{k} d_i^2/n$. The proportion to the total variation is

$$\lambda_k = \frac{\sum_{i=1}^{k} d_i^2}{\sum_{j=1}^{p} d_j^2} \tag{3}$$

If there is a small k such that $\lambda_k \approx 1$, the dimension of the residual image matrix can be reduced from p to k with most variations contained in $\mathbf{M}_k$. Then, we can use $\mathbf{M}_k$ in the next stage analysis.

A. Robust PCA

Traditional PCA has an issue that it is sensitive to large scale outliers. To deal with this issue, research community proposed methods based on outliers removing [12], weighted SVD [13], and robust error function [14]. These methods, however, can not guarantee the optimality of the result. Thus, a method called Robust PCA (RPCA) has been proposed [15]. It aims to recover a low-rank matrix from highly corrupted data. Suppose the observed matrix $\mathbf{M}$ is an $n \times p$ matrix, and it can be decomposed as

$$\mathbf{M} = \mathbf{L}_0 + \mathbf{S}_0 \tag{4}$$

where $\mathbf{L}_0$ is a low-rank matrix and $\mathbf{S}_0$ is a sparse matrix. If $\mathbf{L}_0$ satisfies some incoherent conditions and $\mathbf{S}_0$ is sufficiently sparse, then $\mathbf{L}_0$ and $\mathbf{S}_0$ can be recovered by solving a tractable convex optimization problem:

$$\min \|\mathbf{L}\|_* + \lambda \|\mathbf{S}\|_1, \tag{5}$$

$$\text{s.t. } \mathbf{L} + \mathbf{S} = M, \tag{6}$$

where $\|\mathbf{L}\|_*$ denotes the nuclear norm of matrix $\mathbf{M}$ and $\|\mathbf{S}\|_1$ denotes the ℓ_1-norm of $\mathbf{S}$, λ is a hyperparameter. This problem can be solved via principal component pursuit (PCP) method [15]

III. Methodology

In this section, we introduce our method. It includes the development of regression PCA (RegPCA) in Section III-A, the combination of RegPCA and RPCA in Section III-B, and the implementaion of our method to video and image processing in Section III-C.

A. RegPCA

In image and video processing, the motivation of RegPCA is to account for a given image by a few underlying images. The given image is treated as the response. The underlying images are treated as explanatory variables. In order to fit those by a regression approach, we need to convert all the images into vectors by matrix unfolding methods. After the response is addressed by explanatory variable, a residual vector is derived. The residual vector can be used to reflect the information contained by the given image after the impacts of the underlying images are removed. To carry out a dimension reduction approach, we need to convert the residual vector back to a matrix. Then, we obtained our RegPCA method.

Suppose that a matrix for a response variable has been unfolded to a vector, and a number of matrices for explanatory variables have also been unfolded to vectors. Then, the interest is to study the relationship between a given image for the response and a number of base images for the explanatory variables.

Let the unfolded vector for the given image be $\mathbf{y}$, and the unfolded vectors for the base images be $\mathbf{x}_1, \dots, \mathbf{x}_p$. Then, the relationship is modeled by

$$\mathbf{y} = \mathbf{1}\beta_0 + \sum_{j=1}^{p} \mathbf{x}_j \beta_j + \boldsymbol{\epsilon}, \tag{7}$$

where $\mathbf{1}$ is a vector with all of its components equal to 1, and $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ is the error vector. We assume each image has n pixels, such that we have $\mathbf{y} = (y_1, \dots, y_n)^\top$ and $\mathbf{X}_j = (x_{1j}, \dots, x_{nj})^\top$ for all $j \in \{1, \dots, p\}$. By matrices, (7) can be equivalently expressed as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \tag{8}$$

where $\boldsymbol{\beta} = (\beta_0, \dots, \beta_p)^\top$ and $\mathbf{X} = (\mathbf{1}, \mathbf{x}_1, \dots, \mathbf{x}_p)$ is an $n \times p-$dimensional matrix and $\boldsymbol{\epsilon} \sim N(0, \sigma^2\mathbf{I})$ is an $n-$dimensional error vector. The maximum likelihood estimator(MLE) of σ and $\boldsymbol{\beta}$ is

$$\begin{aligned} \hat{\boldsymbol{\beta}} &= (\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top\mathbf{y} \\ \hat{\sigma}^2 &= \frac{1}{n}(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}})^\top(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}) \end{aligned} \tag{9}$$

In practice, as we are dealing with image, we just use MLE of $\boldsymbol{\beta}$ and calculate $\boldsymbol{\epsilon}$ as $\hat{\boldsymbol{\epsilon}} = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}$

The regression model above offers two parts for the image, linear function $\mathbf{X}\boldsymbol{\beta}$ (i.e. regression part) for the base image and error vector $\boldsymbol{\epsilon}$ (i.e. residual part) for the difference from the base image which would be further analyzed by traditional PCA method.

By folding the $\hat{\boldsymbol{\epsilon}}$ back to image matrix $\mathbf{M}_{\hat{\varepsilon}}$ with size $h \times w$, it is possible to apply PCA into it. In particular, we can set $\mathbf{M} = \mathbf{M}_{\hat{\varepsilon}}$ in (1), where $\mathbf{M}_{\hat{\varepsilon}}$ is a matrix expression of $\hat{\epsilon}$ with the same format as the original image. By SVD, we obtain

$$\mathbf{M}_{\hat{\boldsymbol{\epsilon}}} = \mathbf{U}_{\epsilon}\mathbf{D}_{\epsilon}\mathbf{V}_{\epsilon}^{\top} \tag{10}$$

where $\mathbf{U}_{\epsilon} = (\mathbf{u}_{\epsilon 1}, \ldots, \mathbf{u}_{\epsilon w})$, $\mathbf{V}_{\epsilon} = (\mathbf{v}_{\epsilon 1}, \ldots, \mathbf{v}_{\epsilon w})$, and $\mathbf{D}_{\epsilon} = \mathrm{diag}(d_{\epsilon 1}, \ldots, d_{\epsilon w})$ are defined similarly. For any integer $k \leq w$, we can similarly define post-residual part as:

$$\mathbf{M}_{\hat{\epsilon}k} = \sum_{i=1}^{k} d_{\epsilon i}\mathbf{u}_{\epsilon i}\mathbf{v}_{\epsilon i}^{\top} = \mathbf{U}_{\epsilon k}\mathbf{D}_{\epsilon k}\mathbf{V}_{\epsilon k}^{\top} \tag{11}$$

with $\mathbf{U}_{\epsilon k} = (\mathbf{u}_{\epsilon 1}, \ldots, \mathbf{u}_{\epsilon k})$, $\mathbf{D}_{\epsilon k} = \mathrm{diag}(d_{\epsilon 1}, \ldots, d_{\epsilon k})$, and $\mathbf{V}_{\epsilon k} = (\mathbf{v}_{\epsilon 1}, \ldots, \mathbf{v}_{\epsilon k})$. This can remove the impacts of base image and maintain the information of the different part of the original image.

The RegPCA method can be straightforwardly extended for videos. Note that only one frame is used in the above formulation. If multiple frames are used, then it is a method for videos. To deal with the video, which has multiple frames, the RegPCA method developed in III-A will be applied multiple times. For example, to process a video with k frames, the RegPCA will be applied for k times to process each frame.

If only residual components are considered, then we can also combine RegPCA with other dimension reduction method. For example, we can use RPCA to analyze the residual components given by regression, which provides a combination of RegPCA and RPCA method. This method will be introduced in Section III-B.

B. Combination with RPCA

To combine with RPCA (RegRPCA), the first step is combining the residual components generated by regression step of RegPCA together to form a residual matrix $\mathbf{X}_{\epsilon}$with size $n \times k$ where n is the number of pixel of a frame and k is the number of frames of the video. We treat this $\mathbf{X}_{\epsilon}$ as data matrix in RPCA. And then solve the following optimization problem

$$\begin{gathered}(\mathbf{L}_{\epsilon}, \mathbf{E}_{\epsilon}) = \underset{\mathbf{L}_{\epsilon}, \mathbf{E}_{\epsilon}}{\mathrm{argmin}}\, \|\mathbf{L}_{\epsilon}\|_* + \lambda_0 |\mathbf{E}_{\epsilon}|_1 \\ subject\ to\ \mathbf{L}_{\epsilon} + \mathbf{E}_{\epsilon} = \mathbf{X}_{\epsilon}\end{gathered} \tag{12}$$

where $\|\cdot\|_*$ is nuclear norm which is the sum of singular value of matrix and $\|\cdot\|_1$ is l_1 norm which is the sum of absolute value of elements of matrix. $\mathbf{L}_{\epsilon}$ is low rank part of the residual matrix. $\mathbf{E}_{\epsilon}$ is the sparse part of the residual matrix.

The above optimization problem could be easily solved by PCP method. The low rank part will be used as post-residual for further analysis and the sparse part will be discarded.

Algorithm 1 Video Color Channel Matrices Generation

Input:A video V with k frames, each frame is of size $h \times w$
Output: 3 video color channel matrices $\mathbf{R}, \mathbf{G}, \mathbf{B}$ with size $N \times K, N = h \times w$

```
read V
R = zeros(N, K), G = zeros(N, K),
B = zeros(N, K)
for i in K do
    convert ith frame to image
    convert image to N × 3 matrix m
    m_r = m[:, :, 1], m_g = m[:, :, 2], m_b = m[:, :, 3]
    convert m_r, m_g, m_b into arrays L_r, L_g, L_b with size
  921600 × 1
    R[:, i]=L_r, G[:, i]=L_g, B[:, i]=L_b
Return R, G, B
```

Algorithm 2 Color Channel Matrix Regression

Input:A Video Channel Matrix M
Output: Regression Part $\mathbf{M}_0$ and Residual part $\mathbf{M}_1$ of Video Channel Matrix

```
X = M[:, 1]
M_0 = zeros(N, K)
M_1 = zeros(N, K)
Y = M[:, 2 : K]
for i from 2 to K do
    y = Y[:, i − 1]
    β = Linear_Regression(y, X)
    M_0[:, i] = βX
    M_1[:, i] = y − M_0[:, i]
Return M_0, M_1
```

C. Implementations in Video Separation

To apply RegRPCA approach to video processing, the first step is to convert videos into matrices. The algorithm to convert a video into color channel matrices is illustrated in Algorithm 1. We first load the video into memory and then convert each frame into a $h \times w \times 3$ matrix where (h, w) represent the width and height of the frame and 3 represents the three color channels. We then separate this matrix into three parts with size $h \times w$ representing R,G,B channel of each frame. Next, we unfold the matrices into three column vectors with size $(h \times w) \times 1$. Finally, we get three video color channel matrices with size $N \times k$ where k is the number of frames of video and $N = h \times w$ is the total number of pixels of one frame.

We treat color channel matrices $\mathbf{R}, \mathbf{G}, \mathbf{B}$ as three data matrices and apply RegPCA or RegRPCA to them respectively. The procedure to create regression part of channel matrix is illustrated in Algorithm 2. The first frame of video is used as the base image and the rest of the frames are regressed against this base image.

In a nutshell, using this two-step RegRPCA approach to separate moving objects from a video works as follows.

After applying regression to each video channel matrix, we can combine the regression parts of each channel matrix, $\mathbf{R}_0, \mathbf{G}_0, \mathbf{B}_0$, to form a video, which represents the background information of the original video. The residual terms of the channel matrices, $\mathbf{R}_1, \mathbf{G}_1, \mathbf{B}_1$, are then processed by RPCA. Integrating the resulting post-residual color matrices of each channel, we are able to create video with foreground only.

D. Complexity Analysis

Since the regression step and PCA step are independent. The complexity of RegPCA C_{RP} (either time or space) can be expressed as $C_{RP} \sim max(C_R, C_P)$. Where C_R is the complexity of regression and C_P is the complexity of PCA. When modification is applied, for example if PCA is replaced by RPCA. then the corresponding C_P is thus the complexity of RPCA.

IV. Experiments

Based on the method, three experiments were conducted to evaluate the proposed RegRPCA. The first experiment is a color separation task on a screen recorded mobile game whose light intensity and camera position were totally fixed [16]. The second and the third experiments are color video separation tasks for real-world videos shot from a fixed camera [17], [18]. Some frames of videos are show in Fig. 1a, Fig. 1b and Fig. 1c. All the algorithms were implemented in Python.

The game video represents an ideal condition that the camera is fixed and the light intensity does not change and moving objects are small. The airport video represents a bit more realistic conditions. It has some small camera jitters and a bit light intensity changes during the recording. The snow train video represents another condition. It has fewer camera jitters and and less light variance than that of the airport video. However, the snow scene has a unique characteristic. The moving objects cover a large area of the screen, which means the residual portion is not sparse.

A. Evaluation Metrics

We use color deviation to evaluate to what extent the color of the resulting images has deviated from the source data.

The color deviation C of a frame is defined as follows:

$$C = \frac{1}{3}\frac{1}{N_{eff}}\sum_{j=0}^{n}\frac{|X_{ij} - X'_{ij}|^2 H(X'_{ij})H(X_{ij})}{X_{ij}^2}, \tag{13}$$

where N_{eff} is the total effect and is defined below.

$$N_{eff} = \sum_{i=1}^{3}\frac{1}{\sum_{j=1}^{n} H(X'_{ij})H(X_{ij})}, \tag{14}$$

where X_{ij} is the ith color channel value of jth pixel of the frame, X'_{ij} is the corresponding value of the residual part of the frame. The Heaviside setp function $H(x)$ is defined as follows:

$$H(x) = \begin{cases} 1 \text{ if } x > 0 \\ 0 \text{ if } x = 0 \end{cases}, \tag{15}$$

Therefore, the color deviation of a video of k frames can be expressed in Equation 16.

$$C_{total} = \frac{1}{k}\sum_{l=1}^{k} C_l, \tag{16}$$

When the value of C_{total} small, it means there are less color deviation from the original videos.

B. Experiment One: Game Video

The first experiment is a video separation task on the game video. The results of color deviation are shown in Table I. Lower deviation value indicates better color retention. RegRPCA with 10 iterations has an average deviation value of 0.727 while RPCA with 10 iterations has an average value of 0.972. Even after 50 iterations, the deviation value is only decreased up to 0.966. Visual performance of the two approaches is illustrated in Fig. 2a. The 1st row is the orgnial video; the 2nd row is RegRPCA; the 3rd row is RPCA with 10 iterations and the last row is RPCA with 50 iterations. The experiment results show that in such condition, the results of RegRPCA has systematically lower color deviation than applying RPCA only. This is also visible when examining the visual performance of the images. The colors of each gaming characters in the results of RegRPCA are similar to the original images. There are some ghost effects in the output images due to matrix reduction. In RPCA, although the shapes of the characters are presented clearly, the colors of the pictures are completely off.

In terms of the accuracy of separation, the visual effects show that both RegRPCA and RPCA can extract the information of moving characters correctly, but the RegRPCA outputs much cleaner image.

C. Experiment Two: Airport Video

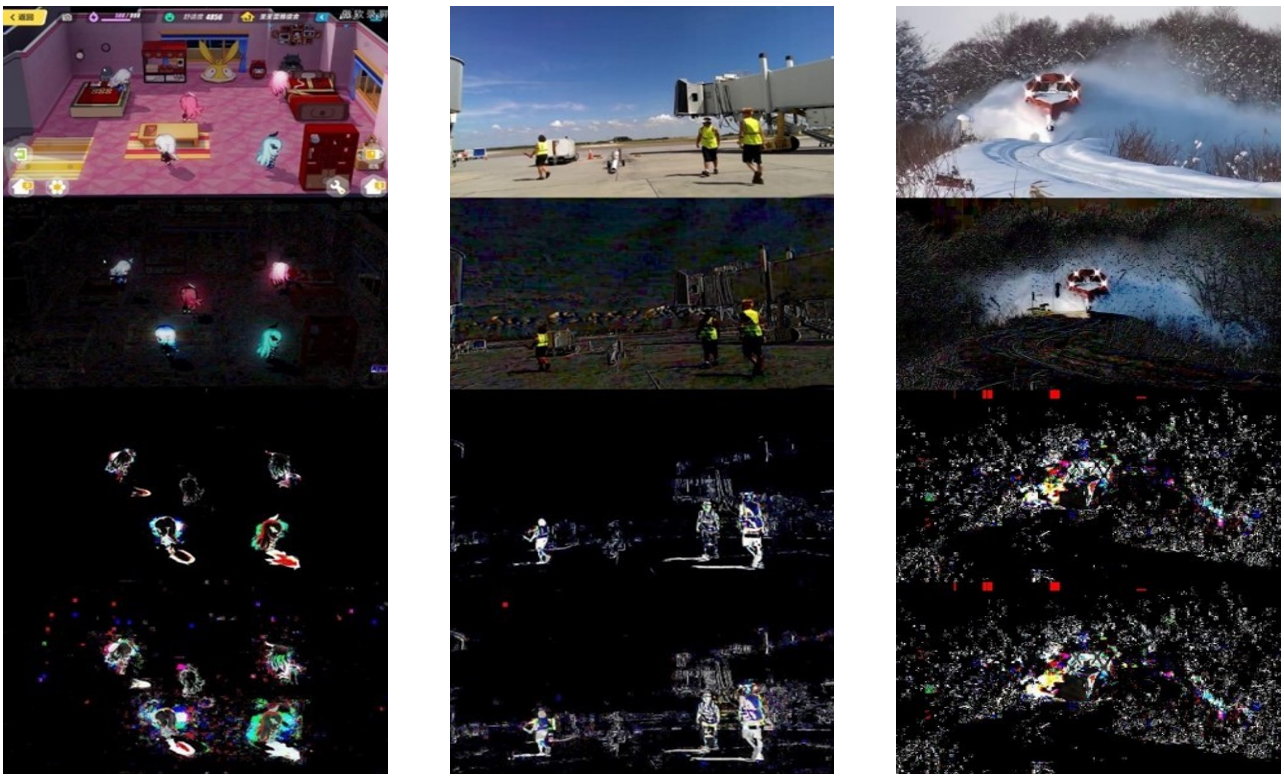
The second experiment is a video separation task on the airport video. The numeric results of color deviation of the video are in Table II. The numeric results showed similar trends as in the game video. RegRPCA has an average deviation of 0.718 while the average of RPCA is around 0.98. Visual inpsection fo the results are shown in Fig. 2b.

The experiment results show that in this video, RegRPCA has less color deviation than RPCA only. Even when the iteration of RPCA goes much higher, the color deviation of RPCA is still higher than RegRPCA. It can also be seen from the results of visual performance. The color of people in the results of RegRPCA is similar to the original images although the background is not clean due to the camera jitters and the process of simple matrix reduction. In RPCA, the shape of moving objects can be figured out quickly. But when the iteration is low (10 to 50), the color of result is very different from the original one.

As for the accuracy of separation, we find that RPCA can extract the shape of people well but not much color information from the results. RegRPCA can extract the information of moving objects well with the correct color information. Part of background information were visible in both cases due to small camera jitters.

(a) Examples of Frames in Game Videos (b) Examples of Frames in Airport Videos (c) Examples of Frames in snow train Videos

Fig. 1: Video Examples



(a) Results of Game Video. (b) Results of Airport Video (c) Results of snow train Video.

Fig. 2: Visual Results. Row 1 are the original frames. Row 2 are the results of RegPCA. Row 3 are the results of RPCA with 10 iterations. Row 4 are the results of RPCA with 50 iterations.

D. *Experiment Three: Snow Train Video*

The third experiment is a video separation task on the snow train video [18]. The color deviation scores of the methods are in Table III and the visual performance is in Fig. 2c.

The numeric deviation results show that in this video, RegRPCA also has much less color deviation at around 0.53. While the deviation results of RPCA is a bit more than 1.02, noticeably higher than the results of RPCA from the other two experiments. This is because that in the Snowtrain video, the moving objects contains snow that spreads out and covers a large area of the screen. The sparse portion of the matric is not sparse enough in such situation for RPCA to be as effective as it was in the previous two experiments. Visual inspection of RPCA (10 iterations and 50 iterations) show poor results with profound noises. The train and the snow are hardly recognizable from the resulting images. The RegRPCA was able to extract the color information and the shape information very well. In short, RPCA can roughly depict the shape of the train but not the snow. RegRPCA handles both the train and the snow very well.

TABLE I: Color Deviation of the Game Video.

method/channel	Red	Green	Blue	Average
RegRPCA (10 iter)	0.7660	0.6877	0.7265	0.7267
RPCA(10 iter)	0.9768	0.9705	0.9687	0.97203
RPCA(50 iter)	0.9747	0.9626	0.9604	0.9659

V. Conclusion and Future Work

A. *Conclusion*

From the results of the above experiments, it is evident that by applying the RegPCA approach to RPCA, foreground

TABLE II: Color Deviation of the Airport Video

method/channel	Red	Green	Blue	Average
RegRPCA (10 iter)	0.6951	0.7402	0.7172	0.7175
RPCA(10 iter)	0.9850	0.9801	0.9838	0.9830
RPCA(50 iter)	0.9731	0.9653	0.9707	0.9697

TABLE III: Color Deviation of the Snowtrain Video

method/channel	Red	Green	Blue	Average
RegRPCA	0.5492	0.5111	0.5322	0.5302
RPCA(10 iter)	1.0297	1.0298	1.0288	1.0295
RPCA(50 iter)	1.0245	1.0249	1.0244	1.0246

object separation tasks were significantly improved when the videos have colors and when the moving objects are relatively large. The successes of these results showed an interesting perspective of this reverse PCR thinking. At a conceptual level, the background of each frame has similar regression terms. Therefore, the signals of moving objects mostly exist in the residual portion of the regression results. By feeding the residual of regression to RPCA, the expression of the moving objects are much more refined. The limitations of applying RegPCA to this use case assumes the background information does not change a lot during a short time frame. These means additional techniques, such as smoothing, should be applied to tackle camera jitters and sudden light intensity changes.

B. Future Work

We embarked a new paradigm in combining regressions with PCA. It seems trivial if you only look at the mechanism of calculation itself. However, the real implications of this reserve PCR approach is far from being well understood. Two potential directions, both the methodology and the technology, should be explored in future work. From the perspective of methodology, it should be noted that regressions and PCAs both have many different modifications. So there are many possible combinations among them, for example, Ridge Regression and Sparse PCA [19], Polynomial Regression and RPCA, and so on. Developing these methods and finding appropriate applications still have a long way to go.

As for technology, there are two things to be explored. First of all, we only applied very simple techniques in this paper when developing the explanatory variables. Likewise, to generate the residual portion, we simply use the reduction of the matrix. More sophisticated methods could have been applied to improve the performance in color video separation task. For example, applying median blurring to the residual can effectively remove the noisy pixels of the result although it will also negatively impact the quality of the images or videos. How to properly apply these techniques to improve the performance needs further study. Second, the generality of the proposed RegRPCA should be investigated. The experiments conducted in this work showed great results, partially due to the fact that the cameras were in fixed positions and the lighting conditions did not change much. We have not yet evaluated the situations when the cameras are also moving. It is therefore necessary to explore if the proposed approach can be applied to broader situations and videos of different characteristics.

In conclusion, much more can be explored in the direction of regression PCA. We remain hopeful that this approach will have some interesting features and valuable use cases in the world of computer vision, big data analysis and statistical machine learning.

REFERENCES

[1] I. T. Jolliffe, "Principal components in regression analysis," in *Principal component analysis*. Springer, 1986, pp. 129–155.

[2] E. Barshan, A. Ghodsi, Z. Azimifar, and M. Z. Jahromi, "Supervised principal component analysis: Visualization, classification and regression on subspaces and submanifolds," *Pattern Recognition*, vol. 44, no. 7, pp. 1357–1371, 2011.

[3] Z.-y. Liu, J.-f. Huang, J.-j. Shi, R.-x. Tao, W. Zhou, and L.-l. Zhang, "Characterizing and estimating rice brown spot disease severity using stepwise regression, principal component regression and partial least-square regression," *Journal of Zhejiang University Science B*, vol. 8, no. 10, pp. 738–744, 2007.

[4] S. A. Abdul-Wahab, C. S. Bakheit, and S. M. Al-Alawi, "Principal component and multiple regression analysis in modelling of ground-level ozone and factors affecting its concentrations," *Environmental Modelling & Software*, vol. 20, no. 10, pp. 1263–1271, 2005.

[5] Y. Tao, H. Shi, B. Song, and S. Tan, "Parallel quality-related dynamic principal component regression method for chemical process monitoring," *Journal of Process Control*, vol. 73, pp. 33–45, 2019.

[6] J. Wright, A. Ganesh, S. Rao, Y. Peng, and Y. Ma, "Robust principal component analysis: Exact recovery of corrupted low-rank matrices via convex optimization," in *Advances in neural information processing systems*, 2009, pp. 2080–2088.

[7] J. Grosek and J. N. Kutz, "Dynamic mode decomposition for real-time background/foreground separation in video," *arXiv preprint arXiv:1404.7592*, 2014.

[8] J. He, L. Balzano, and A. Szlam, "Incremental gradient on the grassmannian for online foreground and background separation in subsampled video," in *2012 IEEE Conference on Computer Vision and Pattern Recognition*. IEEE, 2012, pp. 1568–1575.

[9] B. Moore, C. Gao, and R. R. Nadakuditi, "Panoramic robust pca for foreground-background separation on noisy, free-motion camera video," *IEEE Transactions on Computational Imaging*, 2019.

[10] K. Pearson, "Principal components analysis," *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, vol. 6, no. 2, p. 559, 1901.

[11] H. Hotelling, "Relations between two sets of variates," in *Breakthroughs in statistics*. Springer, 1992, pp. 162–190.

[12] C. Hastings, F. Mosteller, J. W. Tukey, C. P. Winsor *et al.*, "Low moments for small samples: a comparative study of order statistics," *The Annals of Mathematical Statistics*, vol. 18, no. 3, pp. 413–426, 1947.

[13] K. R. Gabriel and S. Zamir, "Lower rank approximation of matrices by least squares with any choice of weights," *Technometrics*, vol. 21, no. 4, pp. 489–498, 1979.

[14] F. De la Torre and M. J. Black, "Robust principal component analysis for computer vision," in *Proceedings Eighth IEEE International Conference on Computer Vision. ICCV 2001*, vol. 1. IEEE, 2001, pp. 362–369.

[15] E. J. Candès, X. Li, Y. Ma, and J. Wright, "Robust principal component analysis?" *Journal of the ACM (JACM)*, vol. 58, no. 3, p. 11, 2011.

[16] mihoyo, 2019. [Online]. Available: https://honkaiimpact3.mihoyo.com/global

[17] B. Spears, "What happens on the ground when you are landing," 2019. [Online]. Available: https://www.youtube.com/watch?v=nRQ7oIu7IYo&t=5s

[18] satsumannoyaji, 2019. [Online]. Available: https://www.youtube.com/watch?v=n3pTSvQSH_E

[19] H. Zou, T. Hastie, and R. Tibshirani, "Sparse principal component analysis," *Journal of computational and graphical statistics*, vol. 15, no. 2, pp. 265–286, 2006.