Purdue-NCKU program

Lecture 6
2^k Factorial Design

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2^k Factorial Design

- Involving k factors
- Each factor has two levels (often labeled $+$ and $-$)
- Factor screening experiment (preliminary study)
- Factors need not be on numeric scale
- Identify important factors and their interactions

- Interaction (of any order) has **ONE** degree of freedom

2^2 Factorial Design

Example:

factor			replicate			
A	B	treatment	1	2	3	mean
$-$	$-$	(1)	28	25	27	80/3
$+$	$-$	a	36	32	32	100/3
$-$	$+$	b	18	19	23	60/3
$+$	$+$	ab	31	30	29	90/3

- $y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk}$
- Let $\bar{y}(A_+)$, $\bar{y}(A_-)$, $\bar{y}(B_+)$ and $\bar{y}(B_-)$ be the level means of A and B.
- Let $\bar{y}(A_-B_-)$, $\bar{y}(A_+B_-)$, $\bar{y}(A_-B_+)$ and $\bar{y}(A_+B_+)$ be the treatment means

Main Effect

Define main effects of A (denoted again by A) as follows:

$$A = m.e.(A) = \bar{y}(A_+) - \bar{y}(A_-)$$

$= \frac{1}{2}(\bar{y}(A_+B_+) + \bar{y}(A_+B_-)) - \frac{1}{2}(\bar{y}(A_-B_+) + \bar{y}(A_-B_-))$
$= \frac{1}{2}(\bar{y}(A_+B_+) + \bar{y}(A_+B_-) - \bar{y}(A_-B_+) - \bar{y}(A_-B_-))$
$= \frac{1}{2}(-\bar{y}(A_-B_-) + \bar{y}(A_+B_-) - \bar{y}(A_-B_+) + \bar{y}(A_+B_+))$
=8.33

- Let C_A=(-1,1,-1,1), a contrast on treatment mean responses, then

$$\text{m.e.}(A)=\frac{1}{2}\widehat{C}_A$$

- Notice that

$$A = m.e.(A) = (\bar{y}(A_+) - \bar{y}_{..}) - (\bar{y}(A_-) - \bar{y}_{..}) = \widehat{\tau}_2 - \widehat{\tau}_1 = 2\widehat{\tau}_2$$

Main effect is defined in a different way than the factorial modeling. But they are connected and equivalent.

Interaction

- Interaction between A and B: does the effect of A depend on the level of B?
- Define interaction between A and B

$$AB = \text{Int}(AB) = \frac{1}{2}(m.e.(A \mid B_+) - m.e.(A \mid B_-))$$

$$= \frac{1}{2}(\bar{y}(A_+ \mid B_+) - \bar{y}(A_- \mid B_+)) - \frac{1}{2}(\bar{y}(A_+ \mid B_-) - \bar{y}(A_- \mid B_-))$$

$$= \tfrac{1}{2}(\bar{y}(A_-B_-) - \bar{y}(A_+B_-) - \bar{y}(A_-B_+) + \bar{y}(A_+B_+))$$

Let $C_{AB} = (1, -1, -1, 1)$, a contrast on treatment means, then

$$AB{=}\text{Int}(AB){=}\tfrac{1}{2}\hat{C}_{AB}$$

- Notice that Int(AB) = $\widehat{\tau\beta}_{22} - \widehat{\tau\beta}_{21}$ = similar interaction factorial effects difference $=2\widehat{\tau\beta}_{22}$

Effects and Contrasts

factor				effect (contrast)			
A	B	total	mean	I	A	B	AB
$-$	$-$	80	80/3	1	-1	-1	1
$+$	$-$	100	100/3	1	1	-1	-1
$-$	$+$	60	60/3	1	-1	1	-1
$+$	$+$	90	90/3	1	1	1	1

- There is a one-to-one correspondence between effects and contrasts, and contrasts can be directly used to estimate the effects.

- For a effect corresponding to contrast $c = (c_1, c_2, \ldots)$ in 2^2 design

$$\text{effect} = \frac{1}{2} \sum_i c_i \bar{y}_i$$

  where i is an index for treatments and the summation is over all treatments.

- Pay attention to the column of the contrast matrix

2^3 Factorial Design

factor				response		
A	B	C	treatment	1	2	total
−	−	−	(1)	-3	-1	-4
+	−	−	a	0	1	1
−	+	−	b	-1	0	-1
+	+	−	ab	2	3	5
−	−	+	c	-1	0	-1
+	−	+	ac	2	1	3
−	+	+	bc	1	1	2
+	+	+	abc	6	5	11

$$y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\beta\gamma)_{jk} + (\tau\gamma)_{ik} + (\tau\beta\gamma)_{ijk} + \epsilon_{ijkl}$$

factorial effects and contrasts

Main effects:

$$A = m.e.(A) = \bar{y}(A_+) - \bar{y}(A_-)$$

$= \frac{1}{4}(\bar{y}(- - -) + \bar{y}(+ - -) - \bar{y}(- + -) + \bar{y}(+ + -) - \bar{y}(- - +)$
$+\bar{y}(+ - +) - \bar{y}(- + +) + \bar{y}(+ + +))$
$=3.00= 2\hat{\tau}_2$
The contrast is (-1,1,-1,1,-1,1,-1,1)

$B : (-1, -1, 1, 1, -1, -1, 1, 1), B = 2.25$
$C : (-1, -1, -1, -1, 1, 1, 1, 1), C = 1.75$

2-factor interactions:
AB: $A \times B$ componentwise, AB=.75=$\widehat{\tau\beta}_{22}$
AC: $A \times C$ componentwise, AC=.25
BC: $B \times C$ componentwise, BC=.50

High order interaction

k-th order interaction means: does the $(k-1)$-th interaction depend on level of the k-th factor

3-factor interaction:

$$ABC = int(ABC) = \frac{1}{2}(int(AB \mid C+) - int(AB \mid C-))$$

$= \frac{1}{4}(-\bar{y}(---) + \bar{y}(+--) + \bar{y}(-+-) - \bar{y}(++-)$
$+\bar{y}(--+) - \bar{y}(+-+) - \bar{y}(-++) + \bar{y}(+++))$
$=.50 = 2\tau\widehat{\beta}\gamma_{222}$

The contrast is (-1,1,1,-1,1,-1,-1,1)= $A \times B \times C$.

Contrasts for Calculating Effects in 2^3 Design

				factorial effects							
A	B	C	treatment	I	A	B	AB	C	AC	BC	ABC
$-$	$-$	$-$	(1)	1	-1	-1	1	-1	1	1	-1
$+$	$-$	$-$	a	1	1	-1	-1	-1	-1	1	1
$-$	$+$	$-$	b	1	-1	1	-1	-1	1	-1	1
$+$	$+$	$-$	ab	1	1	1	1	-1	-1	-1	-1
$-$	$-$	$+$	c	1	-1	-1	1	1	-1	-1	1
$+$	$-$	$+$	ac	1	1	-1	-1	1	1	-1	-1
$-$	$+$	$+$	bc	1	-1	1	-1	1	-1	1	-1
$+$	$+$	$+$	abc	1	1	1	1	1	1	1	1

Estimates:

$$\text{grand mean:} \frac{\sum \bar{y}_{i.}}{2^3}$$

$$\text{effect :} \frac{\sum c_i \bar{y}_{i.}}{2^{3-1}}$$

General 2^k Design

- k factors: A, B, ..., K each with 2 levels (+,−)
- consists of all possible level combinations (2^k treatments) each with n replicates
- Classify factorial effects:

type of effect	label	the number of effects
main effects (of order 1)	A, B, C, ..., K	k
2-factor interactions (of order 2)	AB, AC, ..., JK	$\binom{k}{2}$
3-factor interactions (of order 3)	ABC,ABD,...,IJK	$\binom{k}{3}$
...	...	...
k-factor interaction (of order k)	$ABC \cdots K$	$\binom{k}{k}$

- Each effect (main or interaction) has 1 degree of freedom

  full model (i.e. model consisting of all the effects) has $2^k - 1$ degrees of freedom.

- Error component has $2^k(n-1)$ degrees of freedom.

- One-to-one correspondence between effects and contrasts:

  - For main effect: convert the level column of a factor using $- \Rightarrow -1$ and $+ \Rightarrow 1$

  - For interactions: multiply the contrasts of the main effects of the involved factors, componentwisely.

General 2^k Design: Analysis

- Estimates:

$$\text{grand mean}: \frac{\sum \bar{y}_{i.}}{2^k}$$

  For effect with contrast $C = (c_1, c_2, \ldots, c_{2^k})$, its estimate is

$$\text{effect} = \frac{\sum c_i \bar{y}_i}{2^{(k-1)}}$$

- Variance

$$\text{Var(effect)} = \frac{\sigma^2}{n2^{k-2}}$$

$$\text{S.E.(effect)} = \frac{\text{MSE}}{n2^{k-2}}$$

- C.I. for every factorial effect

$$\text{effect} \pm t_{\alpha/2, 2^k(n-1)} \text{S.E.(effect)}$$

Unreplicated 2^k Design

- $n = 1$
- No degree of freedom left for error component if full model is fitted.
- Same estimation method
- No error sum of squares available, cannot estimate σ^2 and test effects in both the ANOVA and Regression approaches.
- **Approach 1**: pooling high-order interactions
  - Often assume 3 or higher interactions do not occur
  - Pool estimates together for error
  - Warning: may pool significant interaction

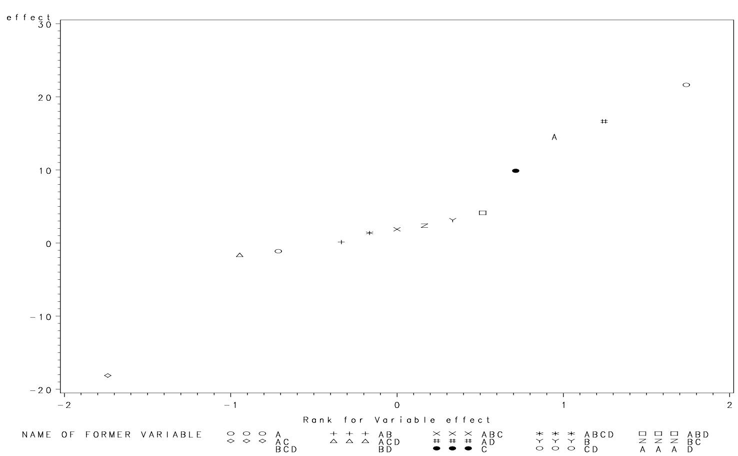
- Approach 2: Using the normal probability plot (QQ plot) to identify significant effects.
  - Recall
    $$\text{Var(effect)} = \frac{\sigma^2}{2^{(k-2)}}$$
    If the effect is not significant (=0), then the effect estimate follows $N(0, \frac{\sigma^2}{2^{(k-2)}})$
  - Assume all effects not significant, their estimates can be considered as a random sample from $N(0, \frac{\sigma^2}{2^{(k-2)}})$
  - QQ plot of the estimates is expected to be a linear line
  - Deviation from a linear line indicates significant effects

A case study

factor				
A	B	C	D	filtration
$-$	$-$	$-$	$-$	45
$+$	$-$	$-$	$-$	71
$-$	$+$	$-$	$-$	48
$+$	$+$	$-$	$-$	65
$-$	$-$	$+$	$-$	68
$+$	$-$	$+$	$-$	60
$-$	$+$	$+$	$-$	80
$+$	$+$	$+$	$-$	65
$-$	$-$	$-$	$+$	43
$+$	$-$	$-$	$+$	100
$-$	$+$	$-$	$+$	45
$+$	$+$	$-$	$+$	104
$-$	$-$	$+$	$+$	75
$+$	$-$	$+$	$+$	86
$-$	$+$	$+$	$+$	70
$+$	$+$	$+$	$+$	96

ALL Ranked Effects

Obs	_NAME_	COL1	effect	neff
1	AC	-9.0625	-18.125	-1.73938
2	BCD	-1.3125	-2.625	-1.24505
3	ACD	-0.8125	-1.625	-0.94578
4	CD	-0.5625	-1.125	-0.71370
5	BD	-0.1875	-0.375	-0.51499
6	AB	0.0625	0.125	-0.33489
7	ABCD	0.6875	1.375	-0.16512
8	ABC	0.9375	1.875	-0.00000
9	BC	1.1875	2.375	0.16512
10	B	1.5625	3.125	0.33489
11	ABD	2.0625	4.125	0.51499
12	C	4.9375	9.875	0.71370
13	D	7.3125	14.625	0.94578
14	AD	8.3125	16.625	1.24505
15	A	10.8125	21.625	1.73938



Effect Selection and Analysis

- Potentially significant effects: A, AD, C, D, AC.
- ANOVA model involving only A, C, D and their interactions (projecting the original unreplicated 2^4 experiment onto a replicated 2^3 experiment)
- Make inferences with non-ZERO MSE
- Diagnostics using residuals.

Fundamental Principles Regarding Factorial Effects

Suppose there are k factors (A,B,...,J,K) in an experiment. All possible factorial effects include

effects of order 1: A, B, ..., K (main effects)

effects of order 2: AB, AC, ...,JK (2-factor interactions)

....

- Hierarchical Ordering principle
  - Lower order effects are more likely to be important than higher order effects.
  - Effects of the same order are equally likely to be important
- Effect Sparsity Principle (Pareto principle)
  - The number of relatively important effects in a factorial experiment is small
- Effect Heredity Principle
  - In order for an interaction to be significant, at least one of its parent factors should be significant.

Fractional Factorials

- May not have sources (time,money,etc) for full factorial design
- Number of runs required for full factorial grows quickly
  - Consider 2^k design
  - If $k = 7 \rightarrow$ 128 runs required
  - Can estimate 127 effects
  - Only 7 df for main effects, 21 for 2-factor interactions
  - the remaining 99 df are for interactions of order ≥ 3
- Often only lower order effects are important
- Full factorial design may not be necessary according to
  - Hierarchical ordering principle
  - Effect Sparsity Principle
- A fraction of the full factorial design (i.e. a subset of all possible level combinations) is sufficient.

There are four factors in the experiment(A, B, C and D), each of 2 levels. Suppose the available resource is enough for conducting 8 runs. 2^4 full factorial design consists of all the 16 level combinations of the four factors. We need to choose half of them.

- If you drop one factors for a 2^3 full factorial design, this factor and their interactions with other factors cannot be investigated.
- Want investigate all 4 factors in the experiment
- A fraction of 2^4 full factorial design will be used.
- Confounding (aliasing) will happen because using a subset

The chosen half is called 2^{4-1} fractional factorial design.

2^{4-1} Fractional Factorial Design

- the number of factors: $k = 4$
- the fraction index: $p = 1$
- the number of runs (level combinations): $N = \frac{2^4}{2^1} = 8$
- Construct 2^{4-1} designs via "confounding" (**aliasing**)
  - select 3 factors (e.g. A, B, C) to form a 2^3 full factorial (basic design)
  - confound (**alias**) D with a high order interaction of A, B and C. For example,

$$D = ABC$$

factorial effects (contrasts)							
I	A	B	C	AB	AC	BC	ABC=D
1	-1	-1	-1	1	1	1	-1
1	1	-1	-1	-1	-1	1	1
1	-1	1	-1	-1	1	-1	1
1	1	1	-1	1	-1	-1	-1
1	-1	-1	1	1	-1	-1	1
1	1	-1	1	-1	1	-1	-1
1	-1	1	1	-1	-1	1	-1
1	1	1	1	1	1	1	1

- Note: 1 corresponds to $+$ and -1 corresponds to $-$.

Verify:
1. the chosen level combinations form a half of the 2^4 design.

2. the product of columns A, B, C and D equals 1, i.e.,

$$I = ABCD$$

which is called the **defining relation**, or $ABCD$ is called a **defining word** (contrast).

Aliasing in 2^{4-1} Design

For four factors A, B, C and D, there are $2^4 - 1$ effects: A, B, C, D, AB, AC, AD, BC, BD, CD, ABC, ABD, ACD, BCD, $ABCD$

Response	I	A	B	C	D	AB	..	CD	ABC	BCD	...	ABCD
y_1	1	-1	-1	-1	-1	1	..	1	-1	-1	...	1
y_2	1	1	-1	-1	1	-1	..	-1	1	1	...	1
y_3	1	-1	1	-1	1	-1	..	-1	1	-1	...	1
y_4	1	1	1	-1	-1	1	..	1	-1	1	...	1
y_5	1	-1	-1	1	1	1	..	1	1	-1	...	1
y_6	1	1	-1	1	-1	-1	..	-1	-1	1	...	1
y_7	1	-1	1	1	-1	-1	..	-1	-1	-1	...	1
y_8	1	1	1	1	1	1	..	1	1	1	...	1

Contrasts for main effects by converting $-$ to -1 and $+$ to 1; contrasts for other effects obtained by multiplication.

$A = \bar{y}_{A+} - \bar{y}_{A-} = \frac{1}{4}(-y_1 + y_2 - y_3 + y_4 - y_5 + y_6 - y_7 + y_8)$
$BCD = \frac{1}{4}(-y_1 + y_2 - y_3 + y_4 - y_5 + y_6 - y_7 + y_8)$

A, BCD are aliases or aliased. The contrast is for A+BCD. A and BCD are not distinguishable.

$AB = \bar{y}_{AB+} - \bar{y}_{AB-} = \frac{1}{4}(y_1 - y_2 - y_3 + y_4 + y_5 - y_6 - y_7 + y_8)$ $CD = \bar{y}_{CD+} - \bar{y}_{CD-} = \frac{1}{4}(y_1 - y_2 - y_3 + y_4 + y_5 - y_6 - y_7 + y_8)$

AB, CD are aliases or aliased. The contrast is for $AB+CD$. AB and CD are not distinguishable.

There are other 5 pairs. They are caused by the defining relation

$$I = ABCD,$$

that is, I (the intercept) and 4-factor interaction $ABCD$ are aliased.

Alias Structure for 2^{4-1} with $I = ABCD$

- Alias Structure:
  $I = ABCD$
  $A = A * I = A * ABCD = BCD$
  $B =$ .... $= ACD$
  $C =$ .... $= ABD$
  $D =$ .... $= ABC$
  $AB = AB * I = AB * ABCD = CD$
  $AC =$ .... $= BD$
  $AD =$ .... $= BC$

- all 16 factorial effects for A, B, C and D are partitioned into 8 groups each with 2 aliased effects.

- When a low order effect is aliased with a high order effect, by Hierarchical Order principle, we tend to believe that the effect is mostly contributed by the low order effect

A Different 2^{4-1} Design

- the defining relation $I = ABD$ generates another 2^{4-1} fractional factorial design, denoted by d_2. Its alias structure is given below.
  $I = ABD$
  $A = BD$
  $B = AD$
  $C = ABCD$
  $D = AB$
  $ABC = CD$
  $ACD = BC$
  $BCD = AC$

- Recall d_1 is defined by $I = ABCD$. Comparing d_1 and d_2, which one we should choose or which one is better?

  1. **Length** of a defining word is defined to be the number of the involved factors.

  2. **Resolution** of a fractioanl factorial design is defined to be the minimum length of the defining words, usually denoted by Roman numbers, III, IV, V, etc...

Resolution and Maximum Resolution Criterion

- d_1: $I = ABCD$ is a resolution IV design denoted by 2^{4-1}_{IV}.
- d_2: $I = ABD$ is a resolution III design denoted by 2^{4-1}_{III}.
- If a design is of resolution R, then none of the i-factor interactions is aliased with any other interaction of order less than $R - i$.

  d_1: main effects are not aliased with other main effects and 2-factor interactions

  d_2: main effects are not aliased with main effects
- d_1 is better, because d_1 has higher resolution than d_2. In fact, d_1 is optimal among all the possible fractional factorial 2^{4-1} designs
- **Maximum Resolution Criterion**
  fractional factorial design with maximum resolution is optimal

How to Analyze 2^{4-1} design

- Compute all effects
- Use QQ plot to determine which ones are significant
- Resolve the ambiguities in aliased effects via the fundamental principles beneficial
- Project the design to a replicated factorial design
- Example
  - I=ABCD
  - QQ plot determine A, B, CD are significant
  - By HO principle C, D are not significant
  - By EH principle, CD are not significant
  - All significant effects are A, B and AB
  - View the data as a 2^2 experiment with 2 replications

General 2^{k-1} Design

- k factors: A, B, ..., K
- can only afford half of all the combinations (2^{k-1})
- Basic design: a 2^{k-1} full factorial for $k-1$ factors: A, B, ..., J.
- The setting of kth factor is determined by alasing K with the $ABC....J$, i.e., $K = ABC \cdots J$
- Defining relation: $I = ABCD....\tilde{I}JK$. Resolution=k
- 2^k factorial effects are partitioned into 2^{k-1} groups each with two aliased effects.
- Use fundamental principles, domain knowledge, follow-up experiment to de-alias.

One Quarter Fraction: 2^{k-2} Design

Parts manufactured in an injection molding process are showing excessive shrinkage. A quality improvement team has decided to use a designed experiment to study the injection molding process so that shrinkage can be reduced. The team decides to investigate six factors
A: mold temperature
B: screw speed
C: holding time
D: cycle time
E: gate size
F: holding pressure
each at two levels, with the objective of learning about main effects and interactions.
They decide to use 16-run fractional factorial design.

- a full factorial has 2^6=64 runs.
- 16-run is one quarter of the full factorial
- How to construct the fraction?

Injection Molding Experiment: 2^{6-2} Design

basic design						
A	B	C	D	$E = ABC$	$F = BCD$	shrinkage
$-$	$-$	$-$	$-$	$-$	$-$	6
$+$	$-$	$-$	$-$	$+$	$-$	10
$-$	$+$	$-$	$-$	$+$	$+$	32
$+$	$+$	$-$	$-$	$-$	$+$	60
$-$	$-$	$+$	$-$	$+$	$+$	4
$+$	$-$	$+$	$-$	$-$	$+$	15
$-$	$+$	$+$	$-$	$-$	$-$	26
$+$	$+$	$+$	$-$	$+$	$-$	60
$-$	$-$	$-$	$+$	$-$	$+$	8
$+$	$-$	$-$	$+$	$+$	$+$	12
$-$	$+$	$-$	$+$	$+$	$-$	34
$+$	$+$	$-$	$+$	$-$	$-$	60
$-$	$-$	$+$	$+$	$+$	$-$	16
$+$	$-$	$+$	$+$	$-$	$-$	5
$-$	$+$	$+$	$+$	$-$	$+$	37
$+$	$+$	$+$	$+$	$+$	$+$	52

Two defining relations are used to generate the columns for E and F.

$$I = ABCE, \text{ and } I = BCDF$$

They induce another defining relation:

$$I = ABCE * BCDF = AB^2C^2DEF = ADEF$$

The complete defining relation:

$$I = ABCE = BCDF = ADEF$$

Defining contrasts subgroup: $\{I, ABCE, BCDF, ADEF\}$

Alias Structure

$I = ABCE = BCDF = ADEF$ implies

$$A = BCE = ABCDF = DEF$$

Similarly, we can derive the other groups of aliased effects.

$A = BCE = DEF = ABCDF$	$AB = CE = ACDF = BDEF$
$B = ACE = CDF = ABDEF$	$AC = BE = ABDF = CDEF$
$C = ABE = BDF = ACDEF$	$AD = EF = BCDE = ABCF$
$D = BCF = AEF = ABCDE$	$AE = BC = DF = ABCDEF$
$E = ABC = ADF = BCDEF$	$AF = DE = BCEF = ABCD$
$F = BCD = ADE = ABCEF$	$BD = CF = ACDE = ABEF$
	$BF = CD = ACEF = ABDE$
$ABD = CDE = ACF = BEF$	
$ACD = BDE = ABF = CEF$	

Wordlength pattern $W = (W_0, W_1, \ldots, W_6)$, where W_i is the number of defining words of length i (i.e., involving i factors)

$$W = (1, 0, 0, 0, 3, 0, 0)$$

Resolution is the smallest i such that $i > 0$ and $W_i > 0$. Hence it is a 2^{6-2}_{IV} design

2^{6-2} **Design: an Alternative**

- Basic Design: A, B, C, D
- $E = ABCD$, $F = ABC$, i.e., $I = ABCDE$, and $I = ABCF$
- which induces: $I = DEF$
- complete defining relation: $I = ABCDE = ABCF = DEF$
- wordlength pattern: $W = (1, 0, 0, 1, 1, 1, 0)$

- Alias structure (ignore effects of order 3 or higher)

$A = ..$	$AB = CF = ..$
$B = ..$	$AC = BF = ..$
$C = ..$	$AD = ..$
$D = EF = ..$	$AE = ..$
$E = DF = ..$	$AF = BC = ..$
$F = DE = ..$	$BD = ..$
	$BE = ..$
	$CD = ..$
	$CE = ..$

- an effect is said to be **clearly estimable** if it is not aliased with main effect or two-factor interactions.

- Which design is better d_1 or d_2? d_1 has six clearly estimable main effects while d_2 has three clearly estimable main effects and six clearly estimable two-factor ints.

Minimum Aberration Criterion

Recall 2^{k-p} with maximum resolution should be preferred. But, it is possible that there are two designs that attain the maximum resolution. How should we further distinguish them?

For example, consider 2^{7-2} fractional factorial design

d_1: basic design: A, B, C, D, E; $F = ABC$, $G = ABDE$
complete defining relation: $I = ABCF = ABDEG = CDEFG$
wordlength pattern: $W = (1, 0, 0, 0, 1, 2, 0, 0)$
Resolution: IV

d_2: basic design: A, B, C, D, E; $F = ABC$, $G = ADE$
complete defining relation: $I = ABCF = ADEG = BCDEFG$
wordlength pattern: $W = (1, 0, 0, 0, 2, 0, 1, 0)$
Resolution: IV

d_1 and d_2, which is better?

Minimum Aberration Criterion

Definition: Let d_1 and d_2 be two 2^{k-p} designs, let r be the smallest **positive** integer such that $W_r(d_1) \neq W_r(d_2)$.

If $W_r(d_1) < W_r(d_2)$, then d_1 is said to have less aberration than d_2.

If there does not exist any other design that has less aberration than d_1, then d_1 has minimum aberration.

Chapter Review

- 2^k design
- 2^k design without replication
- 2^{k-p} design