

HOMEWORK#2

DUE ON 5PM DEC 29

Please email your homework (scanned handwritten solution or typed solution) to my email address with subject "HW 2 of NCKU course"

1. $Y = \text{Bin}(1000, 0.5)$ is a Binomial random variable.
 - (a) What is the EXACT probability that $Y \leq 490$. Note that you only need to write down the formula to compute this probability, while actual calculation is not required
 - (b) By the fact that Y can be expressed as $Y = X_1 + \dots + X_{1000}$ where all X_i 's are i.i.d. Bernoulli random variable with $p = 0.5$, please show that $(Y - 500)/\sqrt{250}$ is approximately a standard normal random variable $N(0, 1^2)$ using the central limit theorem.
 - (c) Prove that approximately, $Pr(\leq 490) =$ the probability of a standard normal random variable $N(0, 1^2)$ being smaller than -0.632
2. The confidence interval for mean of Poisson random variable. Let X_1, \dots, X_n be i.i.d. Poisson random variables $\text{Poi}(\lambda)$.
 - (a) Apply central limit theorem to the sample mean \bar{X} . Note that $E(X) = \text{var}(X) = \lambda$
 - (b) It is known that for standard normal random variable $Pr(-1.96 \leq N(0, 1^2) \leq 1.96) = 0.95$. Combine this fact with solution in (a), and find a 95% confidence interval for λ
3. Consider a linear regression model with one x variable and no intercept, i.e., $Y = \beta X + \epsilon$, where the noise satisfying $\text{var}(\epsilon) = \sigma^2$. Given observations $(x_1, y_1), \dots, (x_n, y_n)$
 - (a) What is the least square estimation of β ? That is to solve the minimization of $\sum (y_i - x_i \beta)^2$ w.r.t. β .
 - (b) What is the mean and variance of your estimation?
 - (c) Given a new $x_{n+1} = 10$. What is your prediction for y_{n+1} ? what is the mean and variance of the prediction?