HOMEWORK#1

Due on 11:59PM Dec 22 (Taiwan Time)

Please email your homework (scanned handwritten solution or typed solution) to my email address qfsong@purdue.edu with subject "HW 1 of NCKU course"

1. Let X be a uniform random variable with probability desity function $f_X(t) = 1$ for $t \in [0, 1]$. Let $Y = \log X$. What is the PDF for random variable Y. Please follow the below steps:

   - What is the cumulative distribution function of X: $F_X(t) = Pr(X \leq t) =$?
   - What is the cumulative distribution function of Y: $F_Y(t) = Pr(Y \leq t) = Pr(\log X \leq t) = Pr(X \leq \exp t) =$?
   - What is the PDF of Y $f_Y(t) = F'_Y(t) =$?

2. The confidence interval for mean of Poisson random variable. Let $X_1, \ldots, X_n$ be i.i.d. Poisson random variables Poi(λ).

   - Apply central limit theorem to the sample mean $\bar{X}$. Note that $E(X) = var(X) = \lambda$
   - It is know that for standard normal random variable $Pr(-1.96 \leq N(0, 1^2) \leq 1.96) = 0.95$. Combine this fact with solution in (a), and find a 95% confidence interval for λ

3. Given **one** observation $X \sim \text{Unif}(\theta, \theta + 1)$, one wants to test $H_0 : \theta \leq 0$ vs $H_1 : \theta > 0$

   - Please design a reasonable reject region
   - For your reject region, please derive the corresponding power function $\beta(\theta)$.

I would like to reschedule the last class on Friday, Dec 26, 6PM-9PM. Please let me know whether the following time work for you or not

- Saturday, Dec 27, 6PM-9PM
- Sunday, Dec 28, 9AM-12PM
- Sunday, Dec 28, 6PM-9PM

Once I confirm the date of class, homework 2 and take-home final exam will be postponed accordingly.