

Final Exam

DUE ON MIDNIGHT, JAN 3

Please email your solution (scanned handwritten solution or typed solution) to my email address with subject "Final exam submission of NCKU course"

1. Let X be non-negative discrete random variable.
 - a. Let a be a positive real value. Show that $\Pr(X \geq a) \leq E(X)/a$.
Hint: Compare $a \times \Pr(X \geq a)$ and $E(X) = \sum_x x \Pr(X = x)$.
 - b. If X is a Poisson random variable with parameter $\lambda = 1$, i.e., $P(X = k) = \frac{e^{-1}}{k!}$ for non-negative integer k . What is the exact probability $\Pr(X \geq 2)$?
 - c. If X is a Poisson random variable with parameter $\lambda = 1$. Please use the result in part a.) to derive an upper bound for $\Pr(X \geq 2)$.

2. An experiment was conducted to assess the yield of a manufacturing process in a chemical factory. Seven 2-level factors (denoted by A, B, \dots, G) are considered in the experiment. It is known that factorial effects of order 3 or higher are negligible. The research decide to design a 2^{7-3} fractional factorial, defined by $E = ABC$, $F = BCD$, $G = ABCD$.

- How many runs (observations) is need for this experiment? Can we have observation for $(A, B, C, D, E, F, G) = (-, +, +, -, -, +, -)$?
- What is the complete defining relation of this design, and what is the wordlength pattern and corresponding resolution of this experiment?
- Please find out the **non-negligible** factorial effects that is aliased with main effect G, and **non-negligible** factorial effects that is aliased with interaction effect DF.
- If the estimated main effect for G is 20, how would you interpret this estimation value?

3. Let X_1 be a random sample from pdf $f(x|\theta) = 2x/\theta^2$, $x \in (0, \theta)$. Construct a $1 - \alpha$ confidence interval for θ based on X_1 .

HINT: X_1/θ has a fixed distribution, i.e., is a pivotal quantity.