

# Final Exam

DUE ON MIDNIGHT, JAN 3

Please email your solution (scanned handwritten solution or typed solution) to my email address with subject "Final exam submission of NCKU course"

1. Let  $X$  be non-negative discrete random variable.
  - a. Let  $a$  be a positive real value. Show that  $Pr(X \geq a) \leq E(X)/a$ .  
Hint: Compare  $a \times Pr(X \geq a)$  and  $E(X) = \sum_x x Pr(X = x)$ .
  - b. If  $X$  is a Poisson random variable with parameter  $\lambda = 1$ , i.e.,  $P(X = k) = \frac{e^{-1}}{k!}$  for non-negative integer  $k$ . What is the exact probability  $Pr(X \geq 2)$ ?
  - c. If  $X$  is a Poisson random variable with parameter  $\lambda = 1$ . Please use the result in part a.) to derive an upper bound for  $Pr(X \geq 2)$ .

2. An experiment was conducted to assess the yield of a manufacturing process in a chemical factory. Seven 2-level factors (denoted by  $A, B, \dots, G$ ) are considered in the experiment. It is known that factorial effects of order 3 or higher are negligible. The research decide to design a  $2^{7-3}$  fractional factorial, defined by  $E = ABC, F = BCD, G = ABCD$ .
- How many runs (observations) is need for this experiment? Can we have observation for  $(A, B, C, D, E, F, G) = (-, +, +, -, -, +, -)$ ?
  - What is the complete defining relation of this design, and what is the wordlength pattern and corresponding resolution of this experiment?
  - Please find out the **non-negligible** factorial effects that is aliased with main effect G, and **non-negligible** factorial effects that is aliased with interaction effect DF.
  - If the estimated main effect for G is 20, how would you interpret this estimation value?

3. Let  $X_1$  be a random sample from pdf  $f(x|\theta) = 2x/\theta^2$ ,  $x \in (0, \theta)$ . Construct a  $1 - \alpha$  confidence interval for  $\theta$  based on  $X_1$ .  
HINT:  $X_1/\theta$  has a fixed distribution, i.e., is a pivotal quantity.