Final Exam

DUE ON US EST JUN 8 MIDNIGHT (TAIWAN TIME: NOON JUN 9)

Please finish 3 questions out of all 4 questions

Please email your solution (scanned handwritten solution or typed solution) to my email address with subject "Final exam submission of NCKU course"

1. An agriculture study tries to find the relationship between crop yield and the usage of fertilizer. Suppose it is already scientifically justified that

The crop yield in **one-acre farm** = $\beta_0 + \beta_1 \cdot (\text{concentration level}(\%) \text{ of fertilizer}) + \epsilon$, (1)

where ϵ is normally distributed with a constant variance σ^2 .

Five farms is used for this experiment to estimate the regression coefficients, and data table is:

| Farm size (acre) | 2 | 3 | 5 | 1 | 4 |
|---------------------------------------|-----|------|------|------|------|
| x-concentration level (%) | 1.0 | 1.25 | 1.50 | 1.75 | 2.00 |
| y-total crop yield in this farm (ton) | 7 | 10 | 17 | 3 | 18 |

a. Given fact (1), what is the distribution of crop yield y of a farm with a size of a acres, given x level concentration of fertilizer? For simplicity, we assume independence among the crop yield of each acre area in this farm.

b. Since the ordinary square estimation is not optimal in this case, please describe how to estimate the parameter β_0 and β_1 via weighted least square. (No actual calculation is required).

- 2. An experiment was conducted to assess the yield of a manufacturing process in a chemical factory. Seven 2-level factors (denoted by A, B, \ldots, G) are considered in the experiment. It is known that factorial effects of order 3 or higher are negligible. The research decide to design a 2^{7-3} fractional factorial, defined by E = ABC, F = BCD, G = ABCD.
 - How many runs (observations) is need for this experiment? Can we have observation for (A, B, C, D, E, F, G) = (-, +, +, -, -, +, -)?

• What is the complete defining relation of this design, and what is the wordlength pattern and corresponding resolution of this experiment?

• Please find out the **non-negligible** factorial effects that is aliased with main effect G, and **non-negligible** factorial effects that is aliased with interaction effect DF.

• If the estimated main effect for G is 20, how would you interpret this estimation value?

3. Let β_1 , β_2 , β_3 and β_4 be the interior angles of a convex quadrilateral, so that $\sum \beta_i = 360$ degrees. Suppose we have available observations Y_i , i = 1, 2, 3, 4 such that $Y_i \sim N(\beta_i; \sigma^2)$ independently, with unknown σ^2 . Can you think of an F-test to test the null hypothesis that the quadrilateral is a rectangle?

Hint 1: view β_4 as $360 - \beta_1 - \beta_2 - \beta_3$ rather than a new parameter. Hint 2: consider the general linear F test, i.e., full model vs reduced model.

4. Let X_1 be a random sample from pdf $f(x|\theta) = 2x/\theta^2$, $x \in (0,\theta)$. Construct a $1 - \alpha$ confidence interval for θ based on X_1 . HINT: θ is a scale parameter, and X_1/θ has a fixed distribution.