Q1

i) α_ℓ is negative.

Typing the same manuscript the second time should be faster. It is the "learning" effect.

ii) $y_{2B} = \gamma + \mu_B + \tau_2 + \varepsilon_{2B}$

$y_{2A} = \gamma + \mu_A + \tau_2 + \alpha_\ell + \varepsilon_{2A}$

$y_{3A} = \gamma + \mu_A + \tau_3 + \varepsilon_{3A}$

$y_{3B} = \gamma + \mu_B + \tau_3 + \alpha_\ell + \varepsilon_{3B}$

$y_{4B} = \gamma + \mu_B + \tau_4 + \varepsilon_{4B}$

$y_{4A} = \gamma + \mu_A + \tau_4 + \alpha_\ell + \varepsilon_{4A}$

$y_{5A} = \gamma + \mu_A + \tau_5 + \varepsilon_{5A}$

$y_{5B} = \gamma + \mu_B + \tau_5 + \alpha_\ell + \varepsilon_{5B}$

$y_{6A} = \gamma + \mu_A + \tau_6 + \varepsilon_{6A}$

$y_{6B} = \gamma + \mu_B + \tau_6 + \alpha_\ell + \varepsilon_{6B}$.

iii) No. $E(\bar{y}_{\cdot A} - \bar{y}_{\cdot B}) = \mu_A - \mu_B + \frac{1}{3}\alpha_\ell$ is biased.

iv). Among all 6 manuscripts, make sure there're $3(A\text{-}B)$ and $3(B\text{-}A)$. So the learning effect cancels out.

Q2 i)

$$\begin{pmatrix} Y_1 \\ \vdots \\ Y_{n_1} \\ Y_{n_1+1} \\ \vdots \\ Y_{n_1+n_2} \end{pmatrix} = \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_{n_1} \\ x_{n_1+1} & 1 \\ \vdots & \vdots \\ x_{n_1+n_2} & 1 \end{pmatrix} \begin{pmatrix} \beta_2 \\ \beta_1 \end{pmatrix} + \vec{\varepsilon}$$

ii)

$$\begin{pmatrix} b_2 \\ b_1 \end{pmatrix} = (X^T X)^{-1} X^T Y = \begin{bmatrix} n_1 + \sum_{i=n_1+1}^{n_1+n_2} x_i^2 & \sum_{i=1}^{n_1+n_2} x_i \\ \sum_{i=1}^{n_1+n_2} x_i & n_2 + \sum_{i=1}^{n_1} x_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{n_1} Y_i + \sum_{i=n_1+1}^{n_1+n_2} X_i Y_i \\ \sum_{i=1}^{n_1} X_i Y_i + \sum_{i=n_1+1}^{n_2+n_1} Y_i \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1}{n_1 + \sum_{i=n_1+1}^{n_1+n_2} x_i^2} & \\ & \dfrac{1}{n_2 + \sum_{i=1}^{n_1} x_i^2} \end{bmatrix} \cdot \begin{pmatrix} \sum_{i=1}^{n_1} y_i + \sum_{i=n_1+1}^{n_1+n_2} x_i y_i \\ \sum_{i=1}^{n_1} x_i y_i + \sum_{i=n_1+1}^{n_1+n_2} y_i \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{\sum_{i=1}^{n_1} y_i + \sum_{i=n_1+1}^{n_1+n_2} x_i y_i}{n_1 + \sum_{i=n_1+1}^{n_1+n_2} x_i^2} \\ \dfrac{\sum_{i=1}^{n_1} x_i y_i + \sum_{i=n_1+1}^{n_1+n_2} y_i}{\sum_{i=1}^{n_1} x_i^2 + n_2} \end{pmatrix}$$

iii) $\begin{pmatrix} b_2 \\ b_1 \end{pmatrix} \sim N\left(\begin{matrix} \beta_2 \\ \beta_1 \end{matrix}, \begin{bmatrix} \dfrac{1}{n_1 + \sum_{i=n_1+1}^{n_1+n_2} x_i^2} & \\ & \dfrac{1}{n_2 + \sum_{i=1}^{n_1} x_i^2} \end{bmatrix} \sigma^2 \right)$

So $b_1 + b_2 \sim N\left(\beta_1 + \beta_2, \left[\dfrac{1}{n_1 + \sum_{i=n_1+1}^{n_1+n_2} x_i^2} + \dfrac{1}{n_2 + \sum_{i=1}^{n_1} x_i^2} \right] \sigma^2 \right)$

3 i) $I = ACD = BCE = ABDE$

ii)

$A = \boxed{CD} = \cdots$
$B = \boxed{CE} = \cdots$ ← not estimatible.
$C = \boxed{AD} = \boxed{BE}$
$D = \boxed{AC} = \cdots$
$E = \boxed{BC} = \cdots$

$AE = BD$.
$AB = DE$

$A = \boxed{CD} = \cdots$ ← not estimatible
$C = \boxed{AD = BE} = \cdots$
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$E = \boxed{BC} = \cdots$

$\boxed{CE} = B = \cdots$
$\boxed{AE} = BD = \cdots$
$\boxed{DE} = AB = \cdots$

↑
estimatible