STAT 514 Homework 7

Due: Oct 30

1. A pork producer is interested in $a = 4$ different chemical treatments to reduce PSE meat. Since each animal carcass can only be split in half ($k = 2$), generate the blocks necessary for this experiment assuming there will be a total of six blocks. What is the name of this design in this situation?
   solution:
   It is a BIBD design with $a = 4, b = 6, k = 2$.

2. An engineer is studying the mileage performance characteristics of 5 types of gasoline additives. In the road test he wishes to use cars as blocks; however, because of a time constraint, he must use an incomplete block design. He runs the balanced design with the five blocks that follow.

	car				
additive	1	2	3	4	5
1		17	14	13	12
2	14	14		13	10
3	12		13	12	9
4	13	11	11	12	
5	11	12	10		8

   (a) Verify that this is a balanced incomplete block design.
   solution:
   We have $a = 5$ treatments (gasoline additives) and $b = 5$ blocks (cars) with $a \leq b$. Each block contains $k = 4$ treatments, each treatment appears in $r = 4$ blocks, and each pair of treatments appears in the same blocks $\lambda = 3$ times. The total number of runs $N = ar = bk = 20$, and $\lambda(a-1) = r(k-1) = 12$. Hence this is a balanced incomplete block design.

   (b) Test if there is a difference between the five additives? draw your conclusions using $\alpha = 5\%$.
   solution:
   The ANOVA table from SAS is as follows (line for Model SS replaced by Type I SS of the block and the treatment)

```
                        Sum of
Source             DF   Squares        Mean Square  F Value Pr > F
car                4    31.20000000    7.80000000   8.57    0.0022
```

trt	4	35.73333333	8.93333333	9.81	0.0012
Error	11	10.01666667	0.91060606		
Corrected Total	19	76.95000000			

Since the p-value for treatment effect is very small (= 0.0012), I conclude that there is a difference between the five gasoline additives.

(c) Obtain the estimates of treatment means (i.e., the adjusted means or the least square means).

solution:

The overall mean is $\bar{y}_{..} = 12.05$, and compute $Q_i = y_{i.} - \frac{1}{k}\sum_j n_{ij}y_{.y}$, where n_{ij} equals 1 if treatment i appears in block j and 0 otherwise. It leads to

$$
\begin{aligned}
Q_1 =& y_{1.} - \frac{1}{4}\sum_{j=1}^{5} n_{1j}y_{.j} = 8.25 \\
Q_2 =& y_{2.} - \frac{1}{4}\sum_{j=1}^{5} n_{2j}y_{.j} = 2.75 \\
Q_3 =& y_{3.} - \frac{1}{4}\sum_{j=1}^{5} n_{3j}y_{.j} = -0.75 \\
Q_4 =& y_{4.} - \frac{1}{4}\sum_{j=1}^{5} n_{4j}y_{.j} = -3.5 \\
Q_5 =& y_{5.} - \frac{1}{4}\sum_{j=1}^{5} n_{5j}y_{.j} = -6.75
\end{aligned}
$$

Finally, the adjusted means are calculated by $\hat{\mu}_i = \hat{\mu} + \hat{\tau}_i = \bar{y}_{..} + \frac{kQ_i}{\lambda a}$. Hence,

$$
\begin{aligned}
\hat{\mu}_1 &= 12.05 + \frac{4*8.25}{3*5} = 14.25; \\
\hat{\mu}_2 &= 12.05 + \frac{4*2.75}{3*5} = 12.7833; \\
\hat{\mu}_3 &= 12.05 + \frac{4*-0.75}{3*5} = 11.85; \\
\hat{\mu}_4 &= 12.05 + \frac{4*-3.5}{3*5} = 11.1167; \\
\hat{\mu}_5 &= 12.05 + \frac{4*-6.75}{3*5} = 10.25;
\end{aligned}
$$

(d) Calculate the standard error of the difference between two treatment mean estimates (i.e. the standard error of $\tau_i - \tau_j$).

solution:

The standard error of the difference of two treatments is estimated by

$$\sqrt{\frac{2k}{\lambda a}\hat{\sigma}^2} = \sqrt{\frac{2k}{\lambda a}MS_E} = \sqrt{\frac{2*4}{3*5}0.9106} = 0.6969$$

(e) Calculate the critical difference for Tukey's pairwise comparisons and draw the conclusions. Are they consistent with the results from SAS with the options

```
lsmeans trt / pdiff adjust = tukey;
```

solution:

$$CD = \frac{q_{\alpha,a,ar-a-b+1}}{\sqrt{2}}\sqrt{\frac{2k}{\lambda a}MS_E} = \frac{4.58}{\sqrt{2}}0.6969 = 2.2569$$

Hence, The pairs of gasoline additives which have significantly different mileage performances are (1, 3), (1, 4), (1, 5), (2, 5). The mileage performances within any other pairs of gasoline additives are not significantly different.
The results from SAS with the options "lsmeans trt / tdiff adjust=tukey;" are given below.

```
          Least Squares Means
Adjustment for Multiple Comparisons: Tukey-Kramer
                      LSMEAN
trt     y LSMEAN      Number
1       14.2500000    1
2       12.7833333    2
3       11.8500000    3
4       11.1166667    4
5       10.2500000    5

         Least Squares Means for Effect trt
    t for H0: LSMean(i)=LSMean(j) / Pr > |t|
           Dependent Variable: y
i/j  1          2          3          4          5
1               2.104587   3.443869   4.496162   5.739781
                0.2838     0.0355     0.0065     0.0010
2   -2.10459               1.339282   2.391576   3.635195
    0.2838                 0.6746     0.1884     0.0259
3   -3.44387   -1.33928               1.052293   2.295913
    0.0355     0.6746                 0.8262     0.2167
4   -4.49616   -2.39158   -1.05229               1.243619
    0.0065     0.1884     0.8262                 0.7280
5   -5.73978   -3.63519   -2.29591   -1.24362
    0.0010     0.0259     0.2167     0.7280
```

In the SAS output, the pairs that are significant different have p-values (the bottom values for entries in the output table) less than 0.05. Hence, the significantly different pairs from SAS are (1, 3), (1, 4), (1, 5), (2, 5), which are consistent with my conclusion.

(f) Suppose the engineer wants to know whether the combination of additives 1 and 2 has the same characteristics as the combination of additives 4 and 5. Use a proper contrast to address this issue and offer your answer.

solution:
The contrast I use is $C = (1, 1, 0, -1, -1)'$. The significance test of it is shown below.

```
Contrast  DF Contrast SS    Mean Square  F Value  Pr > F
C          1  30.10416667   30.10416667  33.06    0.0001
```

Since the p-value for the test is very small (= 0.0001), I conclude that the combination of additives 1 and 2 has significantly different characteristics from the combination of additives 4 and 5.

3. A virologist has asked you to help design an experiment to compare the effects of two different media (A and B) and two different incubation times (12 and 18 hours) on the growth of a specific bacteria. (Since the two factor both have only 2 levels, we could treat it as one factor with 4 levels of treatments.) She has access and money to do 24 different runs (single combination of the two factors like media A for 12 hours) but can only do as many as 6 runs a day.

   (a) Based on this information, propose two potential **balanced** designs that use all 24 runs and present an ANOVA table with sources and degrees of freedom.
   solution:
   I will answer this in terms of 4 trts rather than two sets of two treatments. A RCBD or BIBD could be used here. For the RCBD to be balanced, we utilize only four of the six possible runs each day and run the experiment over 6 days.

Source	DF
Day	5
Trt	3
Error	15
Total	23

   For the BIBD, there are several ways to do this. If only two combinations are run each day, a total of 6 days are needed for the design to be balanced. Since 6 × 2 = 12, the combinations each day could be replicated or you could run the design over 12 days. Likewise, one could run three of the four treatment combinations each day. A total of 4 days are needed for the design to be balanced. Given there are six possible runs per day, each of these combinations could be replicated or run over a total of 8 days. Since more days takes away degrees of freedom from error, the replication design will be used. The ANOVA table below is for a BIBD with $k = 3$.

Source	DF
Day	3
Trt	3
Error	17
Total	23

(b) Suppose the experimenter wants to minimize the length of a 95% confidence interval for a treatment difference ($\mu_i - \mu_j$). Which of the two designs proposed in (a) is better? Explain.

solution:

The std deviation of a treatment difference when using a RCBD is $\sqrt{2\sigma^2/b} = \sqrt{2\sigma^2/6}$. With 15 degrees of freedom, the t-statistic is 2.131. Thus the half-length of the confidence interval would be $2.131\sqrt{\sigma^2/3}$. For the BIBD above, the std deviation of a treatment difference is $\sqrt{2k\sigma^2/(\lambda a)} = \sqrt{2(3)\sigma^2/(4(4))}$. With 17 degrees of freedom the t statistic is 2.110 so the half length is $2.110\sqrt{3\sigma^2/8}$. In this situation,

$$\frac{2.131\sqrt{\sigma^2/3}}{2.110\sqrt{3\sigma^2/8}} < 1,$$

so the RCBD is better. This is the best BIBD among the ones suggested so all other BIBDs are also worse. Other designs could be formed using only 4 days but they would not be balanced. If the experimenter was more interested in comparing certain treatment combinations then a RCBD could be combined with a PBIB over 4 days.