STAT 514 Homework 5

Due: Oct 9

1. Suppose you performed an ANOVA with $a = 4$ treatments and $n = 5$ observations per treatment. If the $MS_E = 16$ and $\alpha = 0.05$, what would the minimum difference have to be between any two means for you to conclude they were significantly different if

   (a) You performed the LSD comparison procedure?
   solution:

   $$CD = t_{\alpha/2,N-a}\sqrt{2MSE/n} = 5.363$$

   (b) You performed the Bonferroni comparison procedure?
   solution:

   $$CD = t_{\alpha/2m,N-a}\sqrt{2MSE/n} = 7.61,$$

   note that $m = 6$ pairs.

   (c) You performed Tukey's multiple comparison procedure (use Table VII)?
   solution:

   $$CD = \frac{q_\alpha(a, N-a)}{\sqrt{2}}\sqrt{2MSE/n} = 7.245$$

   (d) You performed Scheffe's procedure?
   solution:

   $$CD = \sqrt{(a-1)F_{\alpha,a,N-a}}\sqrt{2MSE/n} = 7.887$$

   (e) Explain the relationship between power and the minimum difference. Also state which of the above four is the most powerful and least powerful comparison procedure.
   solution:
   The smaller the minimum difference, the more power the test has. In this study, the most powerful procedure is LSD, which never controls the overall type I error. Among the other three which can use for comparison for all possible pairs, the most powerful comparison procedure is Tukey's method . Scheffe's procedure should be the most conservative one.

2. A clay tile company is interested in studying the effect of cooling temperature on strength. The company has five ovens which produce the tiles, four tiles were baked in each oven and then randomly assigned to one of the four cooling temperatures. The data are shown below.

Cooling Temp	Oven 1	2	3	4	5	mean
5°	3	10	7	4	3	5.4
10°	3	8	12	2	4	5.8
15°	9	13	15	3	10	10
20°	7	12	9	8	13	9.8
Mean	5.50	10.75	10.75	4.25	7.50	7.75

(a) Which type of design was employed? Describe how the fundamental principles of experimental design were followed in this design.
solution:
It followed a randomized complete block design (RCBD). Blocking strategy was applied to control the nuisance factor (ovens), and treatments (cooling temperatures) were randomly chosen for the four tiles inside each block to control for any potential factors affecting the strength.

(b) If $MS_E = 6.275$, compute the F-statistic to determine if there is a difference among the four cooling temperatures (use α= 5%).
solution:
$SS_{trt} = b\sum_{i=1}^{a}(\bar{y}_{i.} - \bar{y}_{..})^2 = 92.95$, thus

$$F = \frac{SS_{trt}/(a-1)}{MS_E} = 4.938,$$

where is greater than the $F_{0.05,3,12}$, hence concludes that there are differences among the four cooling temperatures.

(c) Estimate the relative effiency, and interpret your result.
solution:
$SS_{blk} = a\sum_{j=1}^{b}(\bar{y}_{.j} - \bar{y}_{..})^2 = 141.5$

$$R.E. = \frac{SS_{blk} + b(a-1)MS_E}{(ab-1)MS_E} = 1.98$$

In order for CRD design to achieve that same accuracy as in the RCBD experiment, it will need 1.98 times oberservations compared to RCBD design.

(d) If there is a difference among the four temperatures, perform pairwise comparisons using Tukey's procedure, please calculate by hands first, then use SAS to verify your calculations.
solution:

$$CD = \frac{q_\alpha(a,(a-1)(b-1))}{\sqrt{2}}\sqrt{2MSE/b} = 4.705$$

The maximum difference among the four temperature groups is 10-5.40 = 4.60, which is less than the above critical difference. Hence Tukey's method detects no significant differences among the four temperature groups. As shown below, my computation is consistent with the SAS output.

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 Tukey's Studentized Range (HSD) Test for y
NOTE: This test controls the Type I experimentwise error rate, but it
generally has a higher Type II error rate than REGWQ.
Alpha                                       0.05
Error Degrees of Freedom                    12
Error Mean Square                           6.275
Critical Value of Studentized Range         4.19852
Minimum Significant Difference              4.7035


Means with the same letter are not significantly different.
Tukey Grouping    Mean    N   tmp
A                 10.000  5   3
A                 9.800   5   4
A                 5.800   5   2
A                 5.400   5   1
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(e) Suppose the company believes there is a jump in the strength at 12.5° but otherwise cooling temperature has no effect, that is, 5° and 10° are not different, neither are 15° and 20° , but these two groups of temperatures have different effects. Find a set of orthogonal contrasts that would allow you to test this.
solution:
The three constrast are $C_1 = (1, -1, 0, 0)$, $C_2 = (0, 0, 1, -1)$ and $C_3 = (1, 1, -1, -1)$. where C1 is used to test the difference between temperatures 5° and 10° , C2 to test the difference between temperatures 15° and 20° , and C3 to test the difference between lower temperatures (5° , 10°) and higher temperatures (15° , 20°). It's obviously that they form a complete set of orthogonal contrasts.

(f) Test these contrasts using SAS (or by hand). State your conclusions.
solution:
The SAS output for testing the above contrasts is shown below.

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
C1	1	0.40000000	0.40000000	0.06	0.8049
C2	1	0.10000000	0.10000000	0.02	0.9016
C3	1	92.45000000	92.45000000	14.73	0.0024

Contrasts C1 and C2 are not significant, while contrast C3 is significant. This confirms the belief of the company that there is a jump in the strength at some temperature (12.5°) between 10° and 15° .

3. An experiment was designed to study the performance of four different detergents for cleaning clothes. The following "cleanness" readings (higher=cleaner) were obtained with specially designed equipment for three different types of common stains The conclusion from ANOVA is the detergents are different. However one research suspects that it may not be proper to assume an additive model. Use Tukey's test for non-additivity to settle this issue.

	stain 1	stain 2	stain 3
detergent 1	45	43	41
detergent 2	47	46	52
detergent 3	48	50	55
detergent 4	42	37	49

solution:
To do Tukey's Additivity test, I first fit an additive model and obtain the predicted values y_{ij}^2 then add $q_{ij} = y_{ij}^2$ to the predictor set and fit another model. Then Tukey's test is equivalent to the significance test of q. Here is the SAS output. Note that it is different data set from the one given in the lecture.

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Source  DF  Type III SS   Mean Square   F Value   Pr > F
q       1   0.72686149    0.72686149    0.05      0.8265
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Since q is not significant (p-value= 0.82), I conclude that there're no interaction effects between detergent and stain, and the additive model is valid.