STAT 514 Homework 4

1. To study the effects of pesticides on birds, a scientist randomly and equally allocates $N = 65$ chicks to five diets (a control and four with a different pesticide included). After a month each chick's calcium content (mg) in 2 cm length of bone is measured resulting in the following:

		pesticide			
	Control	1	2	3	4
Mean	11.54	11.00	11.42	11.44	11.28
Std Dev.	0.27	0.47	0.31	0.42	0.31

   (a) Construct the ANOVA table (i.e. compute the between and within SS) and test if there appears to be any differences in means (use α=0.01)

   solution:

$$\begin{aligned} a = 5, \quad N &= 65, \quad n_i = 13 \\ SS_{Trt} &= \sum n_i(\bar{y}_{i.} - \bar{y}_{..})^2 = 2.28 \\ SS_E &= \sum (n_i - 1)s_i^2 = 7.95 \\ SS_{Total} &= SS_{Trt} + SS_E = 10.23 \\ F_0 &= (SS_{Trt}/4)/(SS_E/(65-5)) = 4.305 \\ P - value &< 1\% \end{aligned}$$

   Reject the null hypothesis.

   (b) If the research want to guarantee that the test power is larger than .99 when there is at least one pair of treatments that differ by 2σ, how many samples does he need?

   solution:

   n=18.
   $\Phi = \frac{4\sigma^2 n}{2a\sigma^2} = 0.4n$, sample code:

```
data new;
a=5; alpha=.01;
do n=1 to 20;
df = a*(n-1);
nc = 0.4 * n;
```

```
fcut = finv(1-alpha,a-1,df);
beta = probf(fcut,a-1,df,a*nc);
power = 1-beta;
output;
end;
proc print;
var n nc df beta power; run;
```

2. An experiment is conducted to study the impact of hormone on the liver of rat. Two types of hormones (I, II) each with two levels are involved. We consider the following four treatments: (A) Hormone I at high level; (a) Hormone I at low level; (B) Hormone II at high level; (b) Hormone II at low level. Each treatment is applied to six randomly selected rats. The response is the amount of glycogen (in mg) in the liver of a rat after a certain period of time.

Treatment	Response					
A	106	101	120	86	132	97
a	51	98	85	50	111	72
B	103	84	100	83	110	91
b	50	66	61	72	85	60

Suppose we are interested in the following three contrasts:

Comparision	A	a	B	b
Hormone I vs Hormone II	1	1	-1	-1
Low Level vs High Level	1	-1	1	-1
Equivalence of Level	1	-1	-1	1

(a) Use ANOVA to check if there exist differences between the treatments ($\alpha = 5\%$).
solution:
Anova table from sas is:

```
Source           DF   Sum of Squares   Mean Square   F Value   Pr > F
Model            3    6026.83333       2008.94444    6.97      0.0022
Error            20   5767.00000       288.35000
Corrected Total  23   11793.83333
```

The p-value for the F-statistic is 0.0022, much less than 0.05. Hence we reject the hypothesis and conclude that there are treatment differences.

(b) Are these contrasts orthogonal? Why or why not?
solution:
Since the experiment is a balanced design, two contrasts are orthogonal to each other if and only if their inner product is 0. It is easy to verify that the inner products are all 0, hence these are orthogonal contrast.

(c) Compute the single degree of freedom sum of squares (i.e. SS for contrast) and test each null hypothesis. Interpret the results. (NOTE: Be careful if you use a character string variable to denote the treatment levels. The order of the treatments SAS uses in the contrast statement is different than A, a, B, b. To avoid this, please code the treatments as 1, 2, 3 and 4 respectively.)

solution:

The SAS output for contrast sums of squares and contrasts testing is as follows:

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
C1	1	864.000000	864.000000	3.00	0.0989
C2	1	5162.666667	5162.666667	17.90	0.0004
C3	1	0.166667	0.166667	0.00	0.981

From the table, Contrast C1 is close to significant but not significant (p-value = 0.0989), this tells us that the average effect of hormone I and the average effect of hormone II on are not different from each other; Contrast C2 is very significant (p-value = 0.0004), this shows that the average effect for high levels of hormones and the average effect for low levels of hormones are quite different; Contrast C3 is not significant at all (p-value = 0.9811), so the difference between the high-level and low-level of hormone I is the same as that between the high-level and low-level of hormone II.

3. An experiment is run to determine whether four specific firing temperatures have different effects on the density of certain brick. The experiment generates the following data (temperature.dat on the Blackboard).

```
temperature density
1 22.8 1 22.5 1 21.5 1 21.6 1 22.1
2 21.2 2 19.5 2 20.3 2 20.6 2 19.8
3 20.8 3 21   3 22.2 3 21.6 3 20.4
4 23.7 4 23.3 4 22.4 4 22.6 4 22.9
```

where where the temperature levels are 100, 125, 150 and 175 coded as 1, 2, 3 and 4 respectively.

(a) Test if the firing temperatures have different effects? Use $\alpha = .05$.

solution:

The Anova output:

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	20.252	6.75066667	17.55	<.0001
Error	16	6.156	0.38475000		

Thus, we conclude that different temperatures have different effects on the density of brick.

(b) Since temperature is a quantitative factor, the experimenter is also interested in modeling the functional relationship between brick density and temperature. Use

orthogonal contrasts to fit an orthogonal polynomial model. Test if the linear, quadratic and cubic effects are significant ($\alpha = 5\%$).
solution:
From table X, we have the three orthognal polynomial contrasts is (-3,-1,1,3), (1,-1,-1,1) and (-1,3,-3,1). The sas output is given below:

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
C1	1	3.16840000	3.16840000	8.23	0.0111
C2	1	16.20000000	16.20000000	42.11	<.0001
C3	1	0.88360000	0.88360000	2.30	0.1492

The firrst two contrasts are significant but the third one is not. Hence, I conclude that the linear and quadratic effects are significant but the cubic effect is not. So my final orthogonal polynomial model contains only the linear and quadratic effects,

(c) Use the polynomial model obtained in b), which only includes the significant terms, to find the temperature that produces the lowest density.
solution:
The final model is $f(t) = \beta_0 + \beta_1 P_1(t) + \beta_2 P_2(t)$. $\hat{\beta}_0 = \bar{y}_{..} = 21.64$, $\hat{\beta}_1 = 3.56/20 = 0.18$, (where 3.56 is the estimation of first contrast, and 20 is the sum of squares of the first contrast coefficients) $\hat{\beta}_2 = 3.60/4 = 0.9$, (where 3.60 is the estimation of second contrast, and 4 is the sum of squares of the second contrast coefficients) and,

$$P_1(t) = 2\frac{t - 137.5}{25}$$
$$P_2(t) = \left(\frac{t - 137.5}{25}\right)^2 - 1.25;$$

Thus, we have $f(t) = c_1(t - 132.5)^2 + c_2$ for some constant $c_1 > 0$ and c_2, which means the minimum value is taken when $t = 132.5$.