STAT 514 Homework 11

Due: Dec 7 (Optional)

Heat treating is often used to carbonize metal parts, such as gears. The thickness of the carbonized layer is a critical output variable from this process, and it is usually measured by performing a carbon analysis on the gear pitch (the top of the gear tooth). Six factors are to be studied: A=furnace temperature, B=cycle time, C=carbon concentration, D=duration of the carbonizing cycle, E=carbon concentration of the diffuse cycle, and F =duration of the diffuse cycle. Suppose a 2^{6-2} fractional factorial design will be used for the experiment.

Part I: There are several ways to construct a 2^{6-2} design. The general strategy is as follows. First, A, B, C and D form a 24 full factorial design (basic design) . Second, alias E and F with some high order effects of the basic design. Let d_1 denote the design generated by E = AB and F = CD; and d_2 the design generated by E = ABC and F = BCD.

(a) Derive the complete defining relation for d_1 . Based on the complete relation, work out the alias structure of d_1 . What is the resolution of d_1 ? What is its wordlength pattern?

(b) Suppose effects of order three or higher are negligible, that is, they can be assumed to be zero. How many main effects and two-factor interactions in d_1 are clearly estimable?

(c) Repeat (a) and (b) for d_2.

(d) Which design will you choose for the experiment? why? Does d1 has any advantages at all over d_2 ? Explain.

Part II: A quality improvement team has chosen one of the above designs for the experiment. The design matrix and output are given below.

(e) Which design has been used by the team?

(f) Estimate the factorial effects, then generate a QQ plot to identify potentially important effects.

(g) What did these estimates really estimate? (list them for the important effects only)

(h) ssume that effects of order 3 or higher are negligible, list all possible models (with potentially important effects). Can you use the fundamental principles to determine which model is most likely?

Standard order	run order	A	B	C	D	E	F	pitch
1	5	-	-	-	-	-	-	74
2	7	+	-	-	-	+	-	190
3	8	-	+	-	-	+	+	133
4	2	+	+	-	-	-	+	127
5	10	-	-	+	-	+	+	115
6	12	+	-	+	-	-	+	101
7	16	-	+	+	-	-	-	54
8	1	+	+	+	-	+	-	144
9	6	-	-	-	+	-	+	121
10	9	+	-	-	+	+	+	188
11	14	-	+	-	+	+	-	135
12	13	+	+	-	+	-	-	170
13	11	-	-	+	+	+	-	126
14	3	+	-	+	+	-	-	175
15	15	-	+	+	+	-	+	126
16	4	+	+	+	+	+	+	193

(i) Use ANOVA or regression to justify the final model you have chosen.

(j) Use proper residual analyses to check assumptions.